Introduction to
Mathematics Teaching in the
Intermediate Phase

I-MAT 120
## 2020 Edition

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<th>Institution</th>
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INTRODUCTION TO MATHEMATICS TEACHING IN THE INTERMEDIATE PHASE CURRICULUM AND LEARNING GUIDE

BACHELOR OF EDUCATION IN INTERMEDIATE PHASE TEACHING

1. WELCOME TO THE MODULE

Dear SANTS student,

We welcome you to the Introduction to Mathematics Teaching in the Intermediate Phase (I-MAT 120) module that forms part of the Bachelor of Education in Intermediate Phase Teaching programme and wish you success in your studies.

The purpose of the Bachelor of Education in Intermediate Phase Teaching programme is to offer a curriculum that develops teachers who can acquire and eventually articulate focused knowledge, skills and general principles appropriate for Intermediate Phase teaching, as specified in the Revised Policy on the Minimum Requirements for Teacher Education Qualifications (Department of Higher Education and Training, 2015).

The Bachelor of Education (BEd) qualification requires that teachers develop a depth of specialised knowledge, practical competencies (skills) and experience in an Intermediate Phase context. As part of the BEd qualification, you will need to gain experience in applying what you are learning during a period of Workplace Integrated Learning (WIL). This means you will spend some time teaching Intermediate Phase learners in an authentic (real) context.

The BEd qualification programme is aligned with the Revised Policy on the Minimum Requirements for Teacher Education Qualifications, in particular Appendix C of the policy that outlines the Basic Competencies of a Beginner Teacher (Department of Higher Education and Training, 2015, Government Gazette, No. 38487, p. 62).

2. OUTCOMES OF THE PROGRAMME

At the end of the four-year Bachelor of Education Teaching programme, you must demonstrate the following competencies related to your own academic growth and potential to work with Intermediate Phase learners:

- Read, write and speak the language in ways that facilitate your own academic learning.
- Read, write, and speak the language/s of instruction related to Intermediate Phase in ways that facilitate teaching and learning instruction in the classroom.
- Demonstrate competence in communicating effectively, in general and in relation to Intermediate Phase specialised knowledge in order to mediate and facilitate learning.
- Interpret and use basic mathematics and elementary statistics to facilitate your own academic learning and to manage teaching learning and assessment.
• Use information and communications technology (ICT) in daily life and in teaching.
• Explain the contents and purpose of the national curriculum with particular reference to Intermediate Phase.
• Demonstrate skill in planning, designing, and implementing learning programmes that are developmentally appropriate and culturally responsive to Intermediate Phase context.
• Demonstrate competence in identifying and accommodating diversity in the Intermediate Phase classroom, and in the identification of learning and social problems. This includes planning, designing and implementing learning programmes to accommodate diversity.
• Demonstrate an understanding of the theoretical and pedagogical fields of study that influence education and teaching, as well as learning decisions and practices.
• Demonstrate the ability to function responsibly within an education system, an institution and the community in which an institution is located.
• Demonstrate a respect for and commitment to the educator profession.
• Demonstrate an understanding of:
  o The principles underpinning the disciplines for the various learning areas;
  o Pedagogical content knowledge of the learning subjects to be taught;
  o Planning and designing learning opportunities;
  o Resourcing teaching and learning; and
  o Reflecting on teaching.
• Demonstrate competence in observing, assessing and recording learner progress regularly.
• Reflect upon and use assessment results to solve problems and to improve teaching and learning.
• Demonstrate competence in selecting, using and adjusting teaching and learning strategies in ways that meet the needs of both learners and context;
• Demonstrate competence in managing and administering learning environments and supporting learners in ways that promote social justice ideals;
• Conduct yourself responsibly, professionally and ethically in the classroom, the school and the broader community in which the school is located.
• Display a positive work ethic that benefits, enhances and develops the status of the teaching profession and of early childhood education more broadly.

3. PROGRAMME STRUCTURE

The BEd degree is presented on the National Qualifications Framework (NQF) Exit level 7 with minimum total credits of 498, earned over the four years. The table below shows the curriculum implementation plan of the BEd degree you are studying. It also tells you how many credits each module carries. You will also see at which NQF level the study material has been prepared and which modules you need to pass each year.
This four-year programme has been planned to strengthen the competencies you will need as a beginner teacher.

Outline of modules of the BEd (Intermediate Phase Teaching) programme

<table>
<thead>
<tr>
<th>Module name</th>
<th>Code</th>
<th>NQF L</th>
<th>Credits</th>
<th>Module name</th>
<th>Code</th>
<th>NQF L</th>
<th>Credits</th>
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<tbody>
<tr>
<td><strong>YEAR 1</strong></td>
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<td><strong>SEMESTER 1</strong></td>
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<td><strong>SEMESTER 2</strong></td>
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<tr>
<td>Academic Literacy</td>
<td>B-ALI 110</td>
<td>5</td>
<td>10</td>
<td>Critical Literacies for Teachers</td>
<td>B-CLT 120</td>
<td>5</td>
<td>10</td>
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<tr>
<td>Fundamental Mathematics</td>
<td>B-FMA 110</td>
<td>5</td>
<td>10</td>
<td>Introduction to Mathematics Teaching in the Intermediate Phase</td>
<td>I-MAT 120</td>
<td>5</td>
<td>10</td>
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<tr>
<td>Computer Literacy</td>
<td>B-CLI 110</td>
<td>5</td>
<td>10</td>
<td>Professional Studies in the Intermediate Phase 1: Classroom Practice</td>
<td>I-PFS 121</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Education Studies 1: Theories of Child Development</td>
<td>B-EDS 111</td>
<td>5</td>
<td>10</td>
<td>Education Studies 2: Theories of Learning and Teaching</td>
<td>B-EDS 122</td>
<td>6</td>
<td>12</td>
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<tr>
<td>Introduction to the Language and Literacy Landscape in the Intermediate Phase</td>
<td>I-LLL 110</td>
<td>5</td>
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<td>Introduction to Natural Sciences and Technology Teaching in the Intermediate Phase</td>
<td>I-NST 120</td>
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<tr>
<td>Language of Conversational Competence: isiXhosa / isiZulu / Sepedi</td>
<td>C-LCX 120</td>
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<td>C-LCZ 120</td>
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<td>C-LCS 120</td>
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<td><strong>Sub-total credits for Year 1:</strong></td>
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<td><strong>52-62</strong></td>
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<tr>
<td>Workplace Integrated Learning Year 1</td>
<td>I-WIL 101</td>
<td>5</td>
<td>18</td>
<td><strong>Year 1:</strong></td>
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<tr>
<td><strong>SEMESTER 1</strong></td>
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<td><strong>SEMESTER 2</strong></td>
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<tr>
<td>English Home and First Additional Language and Literacy Teaching in the Intermediate Phase 1</td>
<td>I-EHF 211</td>
<td>6</td>
<td>15</td>
<td>I-EHF 222</td>
<td>6</td>
<td>15</td>
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<tr>
<td>Home Language and Literacy Teaching in the Intermediate Phase 1: Afrikaans / isiXhosa / isiZulu / Sepedi</td>
<td>I-HLA 211</td>
<td>6</td>
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<td>I-HLA 222</td>
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<tr>
<td>B-EDS 213</td>
<td>6</td>
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<td>B-EDS 224</td>
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<tr>
<td>Natural Sciences and Technology Teaching in the Intermediate Phase 1</td>
<td>I-NST 211</td>
<td>6</td>
<td>12</td>
<td>Mathematics Teaching in the Intermediate Phase 1</td>
<td>I-MAT 221</td>
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<tr>
<td><strong>Sub-total credits for Year 2:</strong></td>
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<td><strong>60-63</strong></td>
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<tr>
<td>Workplace Integrated Learning Year 2</td>
<td>I-WIL 202</td>
<td>6</td>
<td>20</td>
<td><strong>Year 2:</strong></td>
<td><strong>128-146</strong></td>
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</table>
## INTRODUCTION TO MATHEMATICS TEACHING IN THE INTERMEDIATE PHASE

### CURRICULUM AND LEARNING GUIDE

<table>
<thead>
<tr>
<th>Module name</th>
<th>Code</th>
<th>NQF L</th>
<th>Credits</th>
<th>Module name</th>
<th>Code</th>
<th>NQF L</th>
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<td><strong>YEAR 3</strong></td>
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<td><strong>SEMESTER 1</strong></td>
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<td><strong>SEMESTER 2</strong></td>
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<tr>
<td>Natural Sciences and Technology Teaching in the Intermediate Phase 2</td>
<td>I-NST 312</td>
<td>6</td>
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<td>Natural Sciences and Technology Teaching in the Intermediate Phase 3</td>
<td>I-NST 323</td>
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<td>Mathematics Teaching in the Intermediate Phase 2</td>
<td>I-MAT 312</td>
<td>6</td>
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<td>Mathematics Teaching in the Intermediate Phase 3</td>
<td>I-MAT 323</td>
<td>6</td>
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| **YEAR 4**  |        |       |         | **YEAR 4**  |        |       |         |
| **SEMESTER 1** |        |       |         | **SEMESTER 2** |        |       |         |
| Digital Pedagogies for Teachers | B-DPT 410 | 5     | 10      | Economic and Management Sciences | I-EMS 420 | 5     | 10      |

40-55

40-55

| **Sub-total credits for Year 3:** | 122-125 |
| **Sub-total credits for Year 4:** | **Total credits for programme:** | **498-529** |

Language competencies will be assessed during the course of your programme.

The modules in the programme can be divided into four broad types of learning (Department of Higher Education and Training, 2015, pp. 9-11). Each type of learning develops specific knowledge, values and attitudes, competencies and skills to achieve the overall exit level outcomes of the programme.
The different types of learning are:

**Fundamental learning, which includes student personal and academic development:**
- This type of learning involves academic literacy, critical literacies for teachers, fundamental mathematics, computer literacy and digital pedagogies for teachers.

**Disciplinary learning:**
This learning includes:
- Subject matter knowledge and includes the study of education and its foundations and specific specialised subject matter;
- Knowledge of the child and how the child grows, develops and learns;
- Understanding of the processes of teaching and learning and the articulation between child development and teaching and learning; and
- Understanding of the historical, socio-political, policy and curriculum contexts of education particularly in South Africa.

**Situational learning:**
- Situational learning refers to knowledge of the varied learning situations of learners. This learning involves specifically learning about the context of the learner. These modules are called professional studies.
- Professional Studies focuses on:
  - The complex context of teachers and teaching and learning in general and Intermediate Phase in particular;
  - Multi-faceted and multi-layered positions and roles a teacher occupies; and
  - Relationships between teaching and learning in the context of the school and classroom and specifically the Intermediate Phase classroom.

**Pedagogical learning:**
- This learning includes disciplinary general pedagogic learning knowledge referring to the study of principles, practices and methods of teaching;
- Pedagogic content knowledge includes specialised pedagogic content or subject knowledge which includes how to present concepts, methods, strategies, approaches and rules of a specific discipline when teaching; and
- It also includes tools for implementing teaching and learning and assessment in context.
Types of learning and modules in the BEd (Intermediate Phase Teaching) programme.

<table>
<thead>
<tr>
<th>Types of learning</th>
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<td>Academic Literacy</td>
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<td>Student personal and academic development</td>
<td>Fundamental Mathematics</td>
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<td>Computer Literacy</td>
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<td>Critical Literacies for Teachers</td>
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<td>Digital Pedagogies for Teachers</td>
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<tr>
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<td>Education studies</td>
<td>Education Studies 2: Theories of Learning and Teaching</td>
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<td>Education Studies 3: Curriculum, Pedagogy and Assessment</td>
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<td>Education Studies 4: History of Education and Education Policies</td>
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<td>Education Studies 5: Sociology of Education</td>
</tr>
<tr>
<td><strong>Situational learning</strong></td>
<td>Professional Studies in the Intermediate Phase 1: Classroom Practice</td>
</tr>
<tr>
<td>Professional studies</td>
<td>Professional Studies in the Intermediate Phase 2: School and Classroom Management</td>
</tr>
<tr>
<td></td>
<td>Professional Studies in the Intermediate Phase 4: Teacher Identity and the Profession</td>
</tr>
<tr>
<td><strong>Pedagogical learning</strong></td>
<td><strong>INTERMEDIATE PHASE (IP)</strong></td>
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<tr>
<td>Pedagogy</td>
<td>Introduction to the Language and Literacy Landscape in the IP</td>
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<td>Introduction to Mathematics Teaching in the IP</td>
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<td></td>
<td>Mathematics Teaching in the IP 1, 2, 3 and 4</td>
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<td></td>
<td>Introduction to Natural Sciences and Technology Teaching in the IP</td>
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<td></td>
<td>Natural Sciences and Technology Teaching in the IP 1, 2, 3 and 4</td>
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<td>Economic and Management Sciences</td>
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<tr>
<td><strong>Languages:</strong></td>
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<tr>
<td>English</td>
<td>Home and First Additional Language and Literacy Teaching in the IP 1, 2, 3 and 4</td>
</tr>
<tr>
<td></td>
<td>Choose another (additional) language at Home Language level OR First Additional Language level: Afrikaans, isiXhosa, isiZulu, Sepedi</td>
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<td></td>
<td>Only if Afrikaans is chosen as another language: choose between isiXhosa, isiZulu, Sepedi as Language of Conversational Competence (LoCC)</td>
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<tr>
<td>Afrikaans</td>
<td>Home Language and Literacy Teaching in the IP 1, 2, 3 and 4</td>
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<td>English First Additional Language and Literacy Teaching in the IP 1, 2 and 3</td>
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</tbody>
</table>
Types of learning | Modules
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Choose between isiXhosa, isiZulu, Sepedi as Language of Conversational Competence (LoCC)
**isiXhosa** | Home Language and Literacy Teaching in the IP 1, 2, 3 and 4
English First Additional Language and Literacy Teaching in the IP 1, 2 and 3
**isiZulu** | Home Language and Literacy Teaching in the IP 1, 2, 3 and 4
English First Additional Language and Literacy Teaching in the IP 1, 2 and 3
**Sepedi** | Home Language and Literacy Teaching in the IP 1, 2, 3 and 4
English First Additional Language and Literacy Teaching in the IP 1, 2 and 3

We call these four types of learning, the **knowledge mix** of a module (Department of Higher Education and Training, 2015, p. 11). The level of knowledge for this module is set at level 5 and it carries 10 credits. For every credit you should spend approximately 10 hours mastering the content. You will thus have to spend at least 100 hours studying the I-MAT 120 material and doing the assignments and any assessments.

The knowledge mix of this level 5 module with the related credits is as follows:

- Disciplinary learning, (Study of education and its foundations, 1 credit and Subject knowledge, 5 credits),
- Pedagogical learning, (General pedagogic knowledge, 1 credit and Pedagogic content knowledge, 2 credits), and
- Situational learning with 1 credit.

### 4. PURPOSE OF THIS MODULE

**Purpose**

This module follows on from the Fundamental Mathematics module in which the student’s mathematics knowledge and competence was the focus. It is a conceptual module with the purpose of introducing students to mathematics as a discipline as well as to the theoretical perspectives that underpin mathematics education as a field of study. This module frames the student’s understanding of mathematics, its knowledge structure, and the content of each content area (knowledge domain). Students will be introduced to the mathematics challenges experienced in primary education in South Africa. The Intermediate Phase and Grade 7 mathematics curriculum is introduced. This module will enable students to develop an overview of how to plan, implement, resource, and evaluate mathematics learning programmes as well as assess learning in the Intermediate Phase and Grade 7. The module provides an overview of how to
assess mathematics learning as well as identify and support learners with mathematics barriers to learning.

**Learning Outcomes**
At the end of this module, students will be able to:

- Understand the discipline of mathematics.
- Understand dominant discourses and theoretical perspectives in mathematics education.
- Understand the knowledge structure and knowledge domains of mathematics.
- Analyse the challenges in mathematics teaching and learning in primary schools in South Africa and internationally.
- Understand the Intermediate Phase mathematics curriculum.
- Understand how to plan, implement, resource and evaluate an Intermediate Phase mathematics learning programme.
- Understand how to assess mathematics learning.
- Understand barriers to mathematics learning in the Intermediate Phase and Grade 7.

**Content**
In this module students are introduced to the field of mathematics, its knowledge structure as well as mathematics education as a field of study. Students engage with dominant discourses and theoretical perspectives in mathematics education. They are exposed to mathematics learning and teaching challenges nationally and internationally as well as to the Intermediate Phase mathematics curriculum. The module provides an overview of how to plan, implement, resource and evaluate a mathematics learning programme in the Intermediate Phase as well as understand barriers to mathematics learning. Specifically, the content comprises:

- The discipline of mathematics.
- Mathematics education:
  - Theoretical perspectives.
  - Dominant discourses.
- An introduction to national and international research on mathematics challenges in primary schools.
- Overview of mathematics in the Intermediate Phase and Grade 7:
  - Overview of mathematics knowledge domains.
  - Plan, implement, resource, evaluate mathematics teaching and learning.
  - Overview of assessment in mathematics.
- Overview of barriers to mathematics learning in the Intermediate Phase.

**Competencies**
- Sound subject knowledge;
- Knowledge of how to select, sequence and pace mathematical content;
- Planning and development of responsive learning environments;
• Planning and assessment of and for learning;
• Development and appropriate use of resources;
• Identification and support for learners with barriers to mathematics learning;
• Applied knowledge of classroom practice; and
• Reflection on teaching practice.

5. WORKING THROUGH THE CURRICULUM AND LEARNING GUIDE (CLG)

We developed the CLG to help you master the content through a distance education mode. You will not have full time tutoring or support but the Student Orientation Booklet, accessible at MySANTS, offers guidelines for distance learning. Aspects such as plagiarism are also explained in this booklet. Make use of MySANTS as a support system for any academic queries.

These guidelines will help you to:

• To work consistently throughout the semester;
• To manage your time efficiently;
• To complete assignments on time; and
• To prepare for tests and examinations.

As you read the CLG, draw on your own experiences and the knowledge you already have. The core text and recommended reading texts included in the CLG will also help you to deepen your understanding of the content and concepts you are working through.

In the CLG, you will find a glossary (word list). The word list will help you understand difficult concepts by providing the definitions (meaning) of such words.

You will also find icons (small pictures). The icons indicate the type of activity you must do. If you do each activity as suggested, you ought to advance and consolidate your understanding of the core concepts in the module. You will find a list of the icons used in this CLG on the next page.

Reading and writing activities have been designed to help you make connections with what you already know, master the content and reflect on what you have learnt. Scenarios (situations resembling an authentic (real-life) context) and dialogues provide background to what you are learning. The review / self-assessment questions are based on the learning outcomes.

Doing each activity will help you understand the content. Get a book or file in which you complete all your activities. Write full sentences and always use your own words to show your understanding. Working systematically through each activity, according
to the estimated time for each activity as provided, will also help prepare you for assessments (assignments and the examination).

Try to find other students to work with. It is easier to share ideas and complete activities when working in a **study group**. Doing so, may help you to master the content more easily.

**Commentaries** appear at the bottom of some activities. Commentaries are not answers but rather a reflection to guide your understanding of the activity and to assist you in knowing whether your own answer is appropriate or not. These commentaries alert you to aspects you need to consider when doing the activity.

| **WRITING ACTIVITY** | An activity is designed to help you assess your progress and manage your learning. Sometimes you will have to **define**, **explain**, and/or **interpret** a concept. Scenarios and dialogues are often used to contextualise an activity. They will also help you bridge theory and practice by linking the concept and real life situations. When responding to the activities, use your own words to show your understanding. Do not copy directly from the text of the CLG. At the end of most activities, you will find commentary that aims to guide your thinking and assess how well you have understood the concepts. The activities are numbered for easy reference. |
| **READING ACTIVITY** | Reading activities may require you to read additional material not printed in the Curriculum and Learning Guide. These readings will be either the full text or part of a core or recommended journal article. Journal articles will give you an expanded or alternative view on a concept. You might be required to explain the concept from a different perspective or compare what has been stated in the CLG with what you read in the journal article. |
| **STUDY GROUP DISCUSSION** | All study group discussions or peer activities require preparation BEFORE the discussion. Preparation includes reading and completing activities in writing. Study group discussions are an opportunity for reflection and for you to apply what you have learnt. Sharing your learning experiences may help you to learn with and from each other. Study group discussions can be done in your own study group or at the SANTS academic support sessions. |
Reflection means to think deeply or carefully about something. Reflection activities require you to review critically what you have learnt and link this with your personal experiences or what you have observed during Workplace Integrated Learning (WIL).

Often questions are provided at the end of each unit. These questions are similar to the type of questions that you may be asked in assignments or examinations.

6. SELF-DIRECTED LEARNING

As a distance education student, it is your responsibility to engage with the content and to direct your own learning by managing your time efficiently and effectively.

We designed the following self-directed learning programme template so that you can plan your time carefully and manage your independent learning. The template will also help you to keep to due dates and thus complete the assignments on time. Careful time management and breaking the work up into manageable chunks will help you work through the content without feeling too stressed. Once you have worked through the activities you should be able to contribute to discussions in your study group and during the non-compulsory student academic support sessions with peers and academic tutors.

When completing the template, consider the following:

- This module is offered in the second semester of your first year of study.
- The semester is 15 - 20 weeks long.
- The module carries 10 credits and has been developed for NQF level 5. It should take you about 100 hours to work through this module.
- The 100 hours will be spent reading, studying, and completing the activities in this CLG, as well as the assignments. You will also spend time preparing and writing the examination.
- The estimated time to read for and complete each activity has been suggested.
- You will need 5 to 10 hours to complete each assignment. This means you will need to budget about 20 hours in total.
- You should plan to spend about 10 to 20 hours preparing for the examination in order to be successful.
Plan your studies and keep pace of your progress by completing the template below. It is not divided into specific weeks, but into the number of units in the CLG. Depending on the nature of the content, it is possible to complete two or more units in one week. Sometimes, you may only be able to complete one unit in a week. Use the template as a guide to help you plan and pace yourself as you work through the content, and activities in each unit.

Add dates to the template indicating when you plan to start working through a particular unit. In addition, using a SANTS academic calendar will also assist you to pace your learning. There is also space to indicate the due dates (deadlines) of the assessments.

<table>
<thead>
<tr>
<th>UNIT IN CLG</th>
<th>CONTENT IN CLG</th>
<th>DATE PLANNED</th>
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<td></td>
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<td></td>
<td>Universal challenges in mathematics education</td>
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<td>Assessment in mathematics</td>
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<tr>
<td>ASSIGNMENT 1</td>
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<td>ASSIGNMENT 2</td>
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<tr>
<td>EXAMINATION</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7. **CORE READING**

Core readings are an important part of your studies as you need to refer to these text(s) when answering some of the questions in the activities.

**Unit 1**

   
   **EBSCO Permalink:** http://search.ebscohost.com/login.aspx?direct=true&db=eue&AN=508080865&site=ehost-live

   
   **EBSCO Permalink:** http://search.ebscohost.com/login.aspx?direct=true&db=eue&AN=116144938&site=ehost-live

**Unit 2**

   
   **EBSCO Permalink:** http://search.ebscohost.com/login.aspx?direct=true&db=eue&AN=85723062&site=ehost-live

   

The text(s) for core reading is available on EBSCOhost and/or permalinks are provided. To access the core reading text(s) use the library tab on MySANTS and click on the EBSCOhost link or use the permalink as provided.

8. **RECOMMENDED READING**

As a distance education student, you cannot only rely on your CLG. We recommend that you also study the following sources so that you have broader insight into the study material:

**Unit 1**

   
   **EBSCO Permalink:** http://search.ebscohost.com/login.aspx?direct=true&db=eue&AN=135961227&site=ehost-live


**Unit 2**


**Open Source:**


**EBSCO Permalink:**

You can access the recommended reading texts by using the library tab on MySANTS and then click on the EBSCOhost link or use the direct links as provided.

**9. ASSESSMENT OF THE MODULE**

The SANTS assessment policy is included in the Student Orientation Booklet and is also available on MySANTS. The policy provides information regarding the types of assessment you will need to do. It includes information about progression rules, perusal of marks, or requests for remarking assessments.

In this module, both formative and summative assessments are done over a period of time (continuous assessment). The activities in the Curriculum and Learning Guide (CLG) are varied and are aimed at assisting you with self-directed learning. Reflecting on what you are learning and discussing it in a study group is always helpful through self-assessment. The personal reflection or review is aimed at revision, reinforcement, and self-assessment while informal peer assessment takes place during the group discussions. The following table provides a summary of the assessment for this module:
9.1 Summary of assessment

Summary of assessment

<table>
<thead>
<tr>
<th>TYPES OF ASSESSMENT</th>
<th>FORM OF ASSESSMENT</th>
<th>WEIGHTING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formative assessment</td>
<td>Two written assignments (100 marks each)</td>
<td>60%</td>
</tr>
<tr>
<td>Summative assessment</td>
<td>Examination (100 marks)</td>
<td>40%</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td>100%</td>
</tr>
</tbody>
</table>

9.2 Self-assessment

An activity aimed at self-assessment is included at the end of each unit. Before you complete the self-assessment activity, reflect on what you have learnt in the respective unit. Revise the main concepts and if there is any topic or concept you are unsure about, go back to the relevant unit and revise.

9.3 Assignments

To support you in your self-directed learning and to keep track of your own progress, we will provide guidelines or the memoranda on MySANTS after the assignments have been marked and returned.

In order to demonstrate that you have gained the knowledge, skills, values, and attitudes described in the learning outcomes of the module, you need to do the following:

- Complete and submit each assignment (100 marks) before the due date.
- Submit both assignments that constitute 60% of your final promotion mark to qualify for admission to the examination.

The task brief (specific information regarding what to do and how to prepare for each assignment) will be explained in the assignment itself. These assignments are provided at the beginning of the first semester together with your CLG for this module. The assignments are also available on MySANTS.

9.4 Semester examination

At the end of the semester, you have the opportunity to sit for a formal summative assessment. This includes the following:

- Write a formal examination, out of 100 marks that will constitute 40% of your final promotion mark. Please read the SANTS Assessment Policy that deals with all aspects of the general assessment and the examination policy.
- A minimum of 40% in the examination is required to qualify for a supplementary examination.
10. PLAGIARISM WARNING FOR STUDENTS

Plagiarism is a form of academic misconduct that can lead to educational or disciplinary action and has severe consequences - in some cases civil or criminal prosecution.

You are guilty of plagiarism if you copy from another person’s work (e.g. a book, an article, a website or even another student’s assignment) without acknowledging the source and thereby pretending it is your own work. You would not steal someone’s purse so why steal his/her work or ideas? Submitting any work that you have written but have already used elsewhere (thus not original), is also a form of plagiarism (auto-plagiarism). An example is when you submit the same assignment or a part of it for two different modules.

Avoiding plagiarism by being academically honest is not difficult. Here is what you should do:

- Submit only your own and original work.
- When using another person’s actual words, sentences or paragraphs, Indicate exactly which parts are not your own (even if presented in the CLG). You must do this by referencing in accordance with the Harvard style - a recognised system specified by SANTS, and you must use quotation marks (“...”).
- You must also reference precisely when using another person’s ideas, opinions or theory. You must do so even if you have paraphrased using your own words.
- You must acknowledge any information or images that you have downloaded from the Internet by providing the URL link (web address) and the date on which the item was accessed (downloaded).
- Never allow any student to use or copy any work from you and then to present it as their own.
- Never copy what other students have done to present as your own.
- Prepare original assignments for each module and do not submit the same work for another module.
- Always list any student who contributed to a group assignment. Never submit the work as if only you worked on the assignment.

The Examination Regulations and Procedures policy contains the following in Section 7.10:
Students may not act in a dishonest way with regard to any test or examination assessment, as well as with regard to the completion and/or submission of any other academic task or assignment. Dishonest conduct includes, among other things, plagiarism, as well as the submission of work by a student for the purpose of assessment, when the work in question is, with the exception of group work as decided by the Academic Committee, the work of somebody else either in full or in part, or where the work is the result of collusion between the student and another person or persons.
All cases of suspected plagiarism will be investigated and if you are found guilty, there are serious consequences. Disciplinary action that may result includes:

- You may lose marks for the assignment/activity. Your marks may be reduced by as much as 50%. You may even be given zero.
- The module may be cancelled and you will have to enrol again. This is a great waste of time and money.
- Your registration for that entire year may be cancelled. That means not all the marks you achieved in all the modules you enrolled for will count anything.
- In some cases, prosecutions in courts of law may be instituted.

Plagiarism is considered such a serious academic crime that you are required to sign the standard document (Declaration of Original Work) to every assignment that you submit by either using the assignment booklet or electronic submission. The Declaration of Original Work is printed on the cover of the assignment booklets.
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INTRODUCTION TO MATHEMATICS TEACHING IN THE INTERMEDIATE PHASE

1. INTRODUCTION

In this module, you will focus on the teaching and learning of mathematics in the Intermediate Phase (Grades 4 to 6), as well as Grade 7. It is the first of five modules about mathematics and mathematics teaching in the Intermediate Phase. This module will empower you, as a student teacher, to teach mathematics effectively to Intermediate Phase and Grade 7 learners and to reflect on your own practice.

As a BEd Intermediate Phase student, this module does not only introduce you to mathematics teaching. It provides an overview of mathematics as a discipline, field of study and provides you with some guidance on planning mathematics teaching in the Intermediate Phase and Grade 7. Throughout the module, the emphasis is on developmental progression and theory and practice of mathematics teaching and learning.

Mathematics teachers need to have in-depth knowledge of the content and how to teach this in a way that children will understand. This means that teachers of mathematics should know what to teach, how to teach it and why they teach the way they teach. From this statement, you can conclude that this module strongly focuses on the WHAT, HOW and WHY of mathematics teaching and learning. This module includes the following:

Unit 1, Understanding Mathematics

This unit will introduce you to mathematics as a discipline. This discussion is underpinned by the theory of constructivism and focuses on how learners learn mathematics by actively constructing knowledge. The focus will be on the following theorist, Piaget, Vygotsky and Bruner. You will gain an insight into their views/theories on mathematics teaching and learning. As an Intermediate Phase mathematics teacher, you will be allowed to use these theorist views in the planning and preparation of your lessons. This unit also provides you with insight into the challenges that are experienced internationally and nationally regarding mathematics education in primary schools.

Unit 2, Mathematics Teaching: Overview and Planning

Unit 2 will provide you with an overview of the mathematics curriculum for the Intermediate Phase, and Grade 7. In this unit, you will learn about planning for teaching
mathematics in the Intermediate Phase and Grade 7. You will also be given an opportunity to reflect on the curriculum and its requirements for mathematics teaching, learning and assessment.

Now that you have an idea of what to expect from this module, we look at the structure and outcomes of Module 1.

2. STRUCTURE AND OUTCOMES OF THIS MODULE

This module consists of two units. This module provides you with the opportunity to work towards the outcomes listed below.

UNIT 1
Understanding mathematics

**Outcomes:** At the end of this unit, you should be able to:
- Define mathematics.
- Explain constructivism and the theorist associated with this approach.
- Understand and describe constructivism with reference to mathematics education.
- Identify and explain factors that influence mathematical proficiency, in South Africa and internationally.

UNIT 2
Mathematics teaching: Overview and planning

**Outcomes:** At the end of this unit, you should be able to:
- Use the CAPS document to plan mathematics lessons.
- Explain how to maximise learning in a mathematics classroom.
- Use different assessment strategies to assess Intermediate Phase and Grade 7 learners in mathematics.
### 3. GLOSSARY

Understanding the following terms will help you in your study of this module:

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolutist view</td>
<td>This refers to the idea that reality is <strong>absolute</strong> – the same for everybody, everywhere.</td>
</tr>
<tr>
<td>Accommodation</td>
<td>It is when children have to change their schemas to <strong>accommodate</strong> the new information or knowledge (Powell &amp; Kalina, 2009).</td>
</tr>
<tr>
<td>Artefact</td>
<td>Man-made object, typically one of cultural or historical interest (Oxford University Press, 2019).</td>
</tr>
<tr>
<td>Assimilation</td>
<td>Children bring in new knowledge to their own schemas (Powell &amp; Kalina, 2009).</td>
</tr>
<tr>
<td>Conceptual</td>
<td>Relating to or based on mental concepts (Oxford University Press, 2019).</td>
</tr>
<tr>
<td>Consecutive</td>
<td>Successive – consecutive numbers follow on from each other.</td>
</tr>
<tr>
<td>Culminate</td>
<td>Reach a point of highest development (Oxford University Press, 2019).</td>
</tr>
<tr>
<td>Developmental progression</td>
<td>Sequences of skills and concepts that learners require to establish mathematical knowledge.</td>
</tr>
<tr>
<td>Encrypted</td>
<td>Convert information into a code (Oxford University Press, 2019).</td>
</tr>
<tr>
<td>Envisage</td>
<td>The way something is imagined or expected to be like in the future.</td>
</tr>
<tr>
<td>Estimate</td>
<td>An approximate calculation or judgement of the value, number, quantity or extent of something (Oxford University Press, 2019).</td>
</tr>
<tr>
<td>Equate</td>
<td>To consider one thing to be the same as or equal to another thing (Oxford University Press, 2019).</td>
</tr>
<tr>
<td>Exploration</td>
<td>The search out; to examine, to investigate (Oxford University Press, 2019).</td>
</tr>
</tbody>
</table>
UNIT 1: UNDERSTANDING MATHEMATICS

1. INTRODUCTION

In this unit, you will learn about the discipline of mathematics and what it means. You will also gain insight into how mathematics is learned by referring to Constructivist theorists such as Piaget, Vygotsky and Bruner. The focus on Constructivism will provide you with the importance of viewing mathematics as a social construct, and a product of reflective human activity (Luneta, 2013).

When learning mathematics, a range of challenges may impede learners to become mathematical proficient. Mathematics teachers need to be aware of the challenges that may influence their teaching and the learners’ mathematical proficiency in the Intermediate Phase. In this unit, factors that influence mathematical proficiency will be discussed by referring to challenges experienced in international and national primary schools.
2. STRUCTURE AND LEARNING OUTCOMES OF UNIT 1

Unit 1 provides you with the opportunity to work towards achieving competency in the areas listed below. Unit 1 has two sections.

<table>
<thead>
<tr>
<th>SECTION 1</th>
<th>Mathematics as a discipline</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Learning outcomes:</strong> At the end of this section, you should be able to:</td>
<td></td>
</tr>
<tr>
<td>• Define mathematics.</td>
<td></td>
</tr>
<tr>
<td>• Describe mathematics as a social construct.</td>
<td></td>
</tr>
<tr>
<td>• Explain different theoretical perspectives of how children learn mathematics.</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>SECTION 2</th>
<th>Universal challenges in mathematics education</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Learning outcomes:</strong> At the end of this section, you should be able to:</td>
<td></td>
</tr>
<tr>
<td>• Explain the factors that influence mathematical proficiency.</td>
<td></td>
</tr>
<tr>
<td>• Identify and discuss international and national challenges in mathematics.</td>
<td></td>
</tr>
<tr>
<td>• Gain an understanding of mathematics to develop your proficiency.</td>
<td></td>
</tr>
</tbody>
</table>

**SECTION 1: MATHEMATICS AS A DISCIPLINE**

1. INTRODUCTION

In this section, mathematics is viewed as a human activity that makes it possible for human beings to make sense of mathematics in the world around them. Therefore, mathematics can be viewed as a social construct.

This section will provide you with an overview of what it means to socially construct knowledge in mathematics. This is done by introducing you to theories (sets of ideas) that reinforce the concept of socially constructing knowledge. The theories discussed will strengthen your knowledge and understanding of mathematics. This knowledge will help you plan, organise and teach mathematics effectively.
2. MATHEMATICS AS A SOCIAL CONSTRUCT

Before the 20th century, an absolutist view of mathematics was predominant (Barnes & Venter, 2008). This view accepted that mathematics consists of absolute and unchallengeable truths in which knowledge was certain. Early in the 20th century, several contradictions arose in mathematics, such as in the theory of sets and functions (Barnes & Venter, 2008). A shift has resulted that mathematics was recognised as a product of human inventiveness and human activity, and thus making it a social construct. This, therefore, accepts that language and rules are vital, including the view that mathematical concepts develop and change (Barnes & Venter, 2008).

This shift to emphasise mathematics as a social construct is supported by and demonstrated in the following definition of mathematics provided in the Curriculum and Assessment Policy Statement (CAPS) by the Department of Basic Education (DBE) (2011):

“Mathematics is a language that makes use of symbols and notations to describe numerical, geometric and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem-solving that will contribute in decision-making” (DBE, 2011, p. 8).

In the extract above, the CAPS document refers to mathematics as a language. As a language, mathematics uses numbers, symbols, and images to communicate thoughts, in the same way as the language we speak uses words and sentences to help us communicate ideas (Naudé & Meier, 2014). Therefore, the teaching and learning of mathematics is a process that depends heavily on language. The reason for this is that abstract mathematical ideas are conveyed in the classroom by talking and writing. Sharing ideas with others through discussions strengthens the notion that mathematics is a language and is socially constructed.

Write two paragraphs, based on your own experiences about what it means to do and know mathematics. Think about:

- How you were taught mathematics at school? (Discussions, group work, exploration, teacher-directed, chalk-and-talk, etc.)
- How did you learn mathematics in school? (Discussing with others, repetitive practising problems, memorising, etc.)
• How did these experiences influence your view of mathematics? (I dislike mathematics, because…; I love mathematics, because…; The best way to learn mathematics is…; The best way to teach mathematics is …)

Naudé and Meier (2014) assert that mathematics is how we organise our everyday lives to make sense of what is going on around us. In other words, mathematics tells us how we know what the time is, how we know how far a certain distance is, or how much to pay for groceries. All this is shared by humans through language that we all understand. To actively be involved in communities, we need to understand what numbers, symbols, shapes and images tell us. This helps us to use our knowledge of mathematics in order to tell the time, know distances, pay for products, build or even participate in sports.

How mathematics is viewed (perceived) offers a way of understanding our world. The perception that you have of mathematics may have an impact on how you will teach the subject. These perceptions, however, shape and influence the decisions that you will make about teaching and learning of mathematics in your class. For example, if teachers perceive mathematics as a set of universal truths and rules that must be taught, they are more likely to see their task as transferring information, facts, and skills to learners. How learners learn is also perceived as the ability to reproduce these facts and skills in standardised tests and assessments.

On the other hand, if teachers perceive mathematics as a social construct, a product of reflective human activity (Luneta, 2013), they are more likely to see their task as engaging learners in meaningful mathematical practices. They also view learners’ learning in terms of conceptual development and understanding, and in which they are challenged to use language, concrete objects, written symbols, diagrams, and pictures to describe and explain their mathematical thinking.

This assumes that there are multiple perspectives on how learners learn mathematics and how it should be taught. The following paragraphs describe three cognitive theorists, their theories and perspectives on how learners learn and become proficient in mathematics.

3. THEORETICAL PERSPECTIVES ON HOW CHILDREN LEARN MATHEMATICS

In the Education Studies 2: Theories of Learning and Teaching module (B-EDS 122), you will learn about different learning and teaching theories. In general, a theory aims to provide an explanation (justifiable reason) for the way physical phenomenon in the world works as they do (Mezirow, 2000). A theory is a heuristic for understanding and making meaning of the world and our experiences, actions and sense of being within it. In
this way, a theory indicates what counts as knowledge and comes to shape the particular view we hold of the world, ourselves and others.

In the context of an educational setting, a learning theory expounds on how knowledge (as a set of facts) come to be acquired, processed, and retained during the process of teaching and learning (Mezirow, 2000). In this way, a learning theory includes concepts (ideas) about how learners learn, what they should learn and how teachers can create optimal conditions for effective learning. You will learn more about teaching and learning theories in the module: Education Studies 2: Theories of Learning and Teaching (B-EDS 122). Keeping this in mind, learning theories do not associate with specific teaching strategies. In other words, the Constructivist Theory, as an example, serves to explain the way in which people can view the world from a mathematical perspective and in the context of teaching and learning; how learners gain, process and retain mathematical concepts and ideas (van de Walle, Loven, Karp & Bay-Williams, 2014).

The theory of Constructivism denotes that learning is a process of active engagement whereby learners actively construct their own knowledge and conceptual understanding; learning is an experiential process and learning is both an individual and social process (Peterson & Seligman, 2004). For this reason, learners should have the opportunity to practise and experience mathematics in collaborative/cooperative groups as well as individually. During the process of learning mathematics, learners should participate, be involved with concrete and authentic artefacts (objects), share ideas, and think critically about what they learn (Naudé & Meier, 2014).

In this module the Constructivist Theory will be discussed with particular reference to the key works of Piaget, Vygotsky and Bruner and how these theories inform mathematics learning. Let us start with Piaget’s ideas.

3.1 Piaget

Jean Piaget’s cognitive Constructivist Theory (1953) focused on the individual and how the individual constructs knowledge internally through an intellectual mode. Piaget, a Swiss developmental psychologist, introduced the concepts of assimilation and accommodation to explain the different processes through which people come to understand and learn about the world.
“According to Piaget assimilation is when children bring in new knowledge to their own schemas and accommodation is when children have to change their schemas to ‘accommodate’ the new information or knowledge” (Powell & Kalina, 2009, p. 2).

Piaget uses the term schema to refer to the basic building blocks of intelligent behaviour and how such knowledge is organised. Schemas can be thought of as knowing that each unit relates to a particular aspect of the world, such as objects, actions and abstract concepts (McLeod, 2015).

Piaget has identified four stages of cognitive development from birth to young adulthood:

- Sensorimotor (0 to 2 years),
- Pre-operational (2 to 7 years),
- Concrete operational (7 to 11 years), and
- Formal operational (11 years to adult).

Learners progress through these four stages differently. Piaget believed that every child would experience every stage of thinking in sequence, and one would not be able to develop to the next stage until they understand the previous stage. Most Intermediate Phase learners fall into the concrete operational (Grades 4 and 5) and formal operational stages of development (Grades 6 and 7). The concrete operational stage is vital as it marks the beginning of a learners’ logical and operational thinking.

The key features of the concrete operational stage are displayed in Table 1:
### Table 1: Key features of the concrete operational stage

<table>
<thead>
<tr>
<th>Feature</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conservation</td>
<td>Understanding that something stays the same in quantity even though its appearance changes.</td>
<td><img src="image" alt="Conservation Example" /></td>
</tr>
<tr>
<td>Classification</td>
<td>This involves grouping objects based on a common characteristic, e.g. colour.</td>
<td><img src="image" alt="Classification Example" /></td>
</tr>
<tr>
<td>Seriation</td>
<td>This involves the ability to mentally arrange items along a measurable aspect, for example in height, weight, colour or size.</td>
<td><img src="image" alt="Seriation Example" /></td>
</tr>
<tr>
<td>Transitivity</td>
<td>Ability to arrange objects according to for example low to high or shortest to longest (and vice versa).</td>
<td><img src="image" alt="Transitivity Example" /></td>
</tr>
<tr>
<td>Reversibility</td>
<td>Able to recognise that actions can be reversed and a process takes place according to specific steps.</td>
<td>3 + 4 = 7, 7 − 4 = 3</td>
</tr>
</tbody>
</table>
Decentration

To consider multiple aspects of a situation.

Source: Bellisarion, Mislavsky and Velasquez (2012)

When comparing the volume of juice in three glasses, learners who can decentrate consider both the height and width of the glasses and not only one aspect such as the height.

3.1.1 Concrete operational stage

Between the ages of 7 and 11 (concrete operational stage), learners start using their five senses to distinguish between two or three dimensions at once. For example, learners at the concrete operational stage can notice that the same amount of liquid in different size containers results in different levels based on the width of the container. Additionally, classification and seriation are the two logical operations that develop during this stage (Piaget, 1977) and both are vital for understanding number concepts.

Teachers need to establish a mathematical environment as this enables learners to construct adequate mathematics knowledge. Burns and Silbey (2000) further assert that learners in the concrete operational stage need hands-on experience. This is important as it provides learners with the opportunity to represent a mathematical solution in multiple ways. To do this, resources should be available in the classroom to make abstract ideas concrete. Resources may include pattern blocks, geoboards, tangrams, counters, dice, and any material easily available to them in the classroom such as paper for folding activities and cutting.

3.1.2 Formal operational stage

A learner in the formal operational stage is capable of forming hypotheses and inferring possible consequences, allowing the child to construct his own mathematical thinking (Ojose, 2008). Learners can relate abstract concepts to concrete situations. For example, when learners encounter a mathematical problem, they can solve it by themselves, without having a teacher to refer to a concrete situation/example. In other words, learners are developing their reasoning skills. These skills include clarification, inference, evaluation, and application (Ojose, 2008). It is the teacher’s role to facilitate learning through setting up problems, monitoring learner exploration and negotiating meaning and understanding with the learner.
Figure 2 indicates the characteristics of learners in the formal operational stage of development:

**Figure 2: Characteristics of learners in the formal operational stage**

**Abstract**
- Adolescents think more abstractly than children.
- They can solve abstract concepts such as algebraic equations.

**Idealistic**
- Adolescents often think about what is possible.
- They think about ideal characteristics of themselves, others and the world.

**Logical**
- Adolescents begin to think more like scientists, devising plans to solve problems and systematically testing solutions.
- This type of thinking is called by Piaget as hypothetical-deductive reasoning.

Piaget's theory, concerning mathematics education, can be seen as that **every learner should construct their own knowledge about a mathematical concept**. This is done if the learner engages actively with the environment and manipulates physical objects to make sense of how the concept works.

---

**2**

Read the following extract from Ojose (2008) titled “Applying Piaget’s Theory of Cognitive Development to Mathematics Instruction” in the core reader and answer the questions that follow.

**“Implications of Piaget's Theory**

Critics of Piaget’s work argue that his proposed theory does not offer a complete description of cognitive development (Eggen & Kauchak, 2000). For example, Piaget is criticized for underestimating the abilities of young children. Abstract directions and requirements may cause young children to fail at tasks they can do under simpler conditions (Gelman, Meck, & Merkin, 1986). Piaget has also been criticized for overestimating the abilities of older learners, having implications for both learners and teachers. For example, middle school teachers interpreting Piaget’s work may assume that their students can always think logically in the abstract, yet this is often not the case (Eggen & Kauchak, 2000).

Although not possible to teach cognitive development explicitly, research has demonstrated that it can be accelerated (Zimmerman & Whitehurst, 1979). Piaget believed that the amount of time each child spends in each
stage varies by the environment (Kamii, 1982). All students in a class are not necessarily operating at the same level. Teachers could benefit from understanding the levels at which their students are functioning and should try to ascertain their students’ cognitive levels to adjust their teaching accordingly. By emphasizing methods of reasoning, the teacher provides critical direction so that the child can discover concepts through investigation. The child should be encouraged to self-check, approximate, reflect and reason while the teacher studies the child’s work to better understand his thinking (Piaget, 1970).

The numbers and quantities used to teach the children number should be meaningful to them. Various situations can be set up that encourage mathematical reasoning. For example, a child may be asked to bring enough cups for everybody in the class, without being explicitly told to count. This will require them to compare the number of people to the number of cups needed. Other examples include dividing objects among a group fairly, keeping classroom records like attendance, and voting to make class decisions.

Games are also a good way to acquire an understanding of mathematical principles (Kamii, 1982). For example, the game of musical chairs requires coordination between the set of children and the set of chairs. Scorekeeping in marbles and bowling requires a comparison of quantities and simple arithmetical operations. Comparisons of quantities are required in a guessing game where one child chooses a number between one and ten and another attempts to determine it, being told if his guesses are too high or too low.”

1. Explain the criticism of Piaget’s work.
2. Define each feature of Piaget’s operational stage of development.
3. Explain the implications of each feature for mathematics teaching. Use relevant mathematics examples to illustrate your explanation.
4. Why is it important for learners to develop mathematical reasoning?
5. The author explains that games are a good way to let learners acquire a solid understanding of mathematical principles. Provide an example of an indigenous (original/cultural) game you played as a child. Explain how the game is played (the rules) and what mathematical skills were developed in the game.

Next, the ideas of Vygotsky will be discussed.

### 3.2 Vygotsky

Vygotsky’s perception of learning was influenced by Piaget’s cognitive constructivist theory. In contrast to Piaget’s belief that the construction of knowledge is a personal process, Vygotsky, a Russian psychologist, believed that knowledge is constructed through **social interaction**. He developed his socio-cultural constructivist theory based on research done in this field. This theory takes into account the important role
that society plays in individual development and how learning is largely dependent on the social environment.

![Figure 3: Lev Vygotsky (1896 - 1934)](image)

Source: Teaching Adolescents [n.d.]

Vygotsky believed that learning takes place on three different levels. First when learners engage and interact with other people in their environment. Secondly, learning takes place when learners interact with a More Knowledgeable Other (MKO) (e.g. a teacher) and thirdly, learning occurs in the Zone of Proximal Development (ZPD).

Vygotsky (1978, p. 86) defines the ZPD as “the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance, or collaboration with their more capable peers.” In other words, the ZPD refers to knowledge that a learner is capable of learning with suitable teaching, support and guidance by a more knowledgeable other (MKO). This guidance or support is also referred to as other-regulation and includes both implicit and explicit mediation involving varying levels of assistance (Lantolf & Thorne, 2006, p. 200). The MKO, such as a teacher, should provide learners with opportunities to learn mathematics. They should ensure the all learners have the necessary support and tools and using the ZPD to scaffold the process of learning. Dunphy and Dunphy (2003) divided the ZPD into four stages that explain what happens at the beginning, during and after the teaching of a concept. These stages are displayed in Figure 4 with an explanation that follows.
Stage 1 refers to the assistance and guidance provided by the MKO. At this stage, the teaching starts, and learners have little or no understanding of a concept. In this stage, teaching and intervention occur in a variety of ways, depending on the needs of learners. These interventions may include modelling, coaching, asking questions, demonstration, or any other methods of scaffolding.

Stage 2 is called self-assisted (Mntunjani, Adendorff & Siyepu, 2018). As learners gain an understanding of the new concept, they move into the second stage. At this stage, learners can perform, carry out and make sense of tasks independently. Learners have not fully developed or mastered the concept, but the control and direction of their performance have been passed on to the learner. The learners are confident enough to work through the task by themselves, even if they occasionally make mistakes.

Stage 3 takes place where the performance is fully developed and automatic. Any doubts that the learner may have experienced is absent and the learner has internalised the skills/knowledge necessary to perform the task successfully. In other words, the task can be performed by the learner without intervention or assistance from the MKO (Mntunjani et al., 2018).

Stage 4 is where the de-automatisation of performance leads to going back through the ZPD and starting from stage 1 again. This typically occurs later in life when a learner becomes an adult and loses the ability to perform at a proficient level. The learners must go through the developmental process again to restore knowledge and
skills they have lost. Self-evaluation and continuing education are measures that help to delay de-automatisation.

The class teacher should, however, understand that learners will not necessarily move through these stages at the same time as other learners. It is the teacher’s responsibility to assist the learner’s understanding through the ZPD (Vygotsky, 1978). These stages assist mathematics teachers by allowing them to recognise where their learners are in terms of their existing knowledge and skills, to plan activities to mediate the learning process (Mntunjani et al., 2018).

Acquiring mathematical skills and knowledge does not happen in isolation, but takes place when knowledge is shared between learners and a teacher (Naudé & Meier, 2014). Learners have their own mathematical understandings and beliefs based on their experiences. It is the responsibility of the teacher to guide the learners’ thinking. For instance, when learners engage with the teacher on a mathematical problem, the teacher will guide the learners by using probing questions to get them thinking or allow them to interact with each other in small groups (cooperative/collaborative/peer learning). This will help learners to share their ideas, be critical of the mathematical problem, think logically, and form mathematical ideas and develop conceptual understanding.

Mathematics as a subject can be challenging for some learners, and therefore teachers are vital in assisting learners to become proficient in mathematics. In understanding the concept of ZPD and how it interlinks with learners at different developmental levels, teachers can plan a more strategic approach to group and individual learning. It is therefore essential that teachers provide appropriate support such as social interactions, learning experiences involving resources, games, prompts or cues, as well as instruction (teaching) based on a learner’s past performance and current thinking.

Next, you will learn more about Bruner’s ideas.

### 3.3 Bruner

Jerome Bruner, an American constructivist theorist, regarded learning as an active process in which learners construct knowledge based upon their current and past knowledge and experiences. Like Vygotsky, Bruner believed that children’s learning is promoted through social interactions. Bruner proposed three modes of representations or ways in which knowledge is gained, stored and encrypted in memory (McLeod, 2012).
These three modes of representation, as explained by Luneta (2013, p. 30), are discussed next.

- The **enactive mode** of representation (*action-based*) refers to knowing how to do things without using symbols or words to express that knowledge. In this way, learning takes place through manipulating tangible objects and materials and having multiple opportunities in working with such materials. Knowing how to use a spoon and ride a bicycle are examples of this model. In mathematics the following diagram is an example of an enactive mode of representation which has been created by packing out bottle tops and using operational sign cards:

![Enactive Mode Diagram](image)

- The **iconic mode** of representation (*image-based*) involves using symbols, signs and other images to represent concepts. The symbol does not define the concept but rather symbolises it in the way that a stick figure can stand for a person. In mathematics drawings of circles can represent, for example, the bottle tops as shown below:

![Iconic Mode Example](image)

- The **symbolic mode** of representation (*language-based*) is increasingly important in language or mathematical notations and allow the development and consideration of abstract concepts (Shunk, 2004 cited in Luneta, 2013). It is at this stage where mental manipulation of numerals takes place. In mathematics, the following equation represents a symbolic mode of representation:

\[ 2 + 2 = 4 \]
McLeod (2019) stated that Bruner’s perspective suggests a progression from enactive to iconic to symbolic representation when faced with new material. Bruner believed that support must be given through these three stages via a process he called scaffolding (Eugenio, 2009). Bruner’s concept of scaffolding is similar to Vygotsky’s notion of the zone of proximal development (McLeod, 2012). Both believed that through social interaction with more knowledgeable others (teacher/parent/peer) goals can be achieved.

3

180 minutes

Be prepared for the next student academic support session or meeting with your own study group so that you can actively participate.

1. In your group, tabulate and compare the theoretical perspectives (views) of Piaget, Vygotsky and Bruner on how learners learn.
2. Write two paragraphs to highlight the similarities and differences in their ideas with special reference to mathematics learning.
3. In your group, discuss Vygotsky’s ZPD. Write a paragraph to explain your understanding of ZPD and how this will impact on your role as an Intermediate Phase mathematics teacher. Provide examples to substantiate your answer.
4. In your group, carefully read and discuss the following statement:
   
   Bruner’s concept of scaffolding is very similar to Vygotsky’s concept of the zone of proximal development (McLeod, 2019).

   4.1 In approximately 80 words, explain your understanding of this statement by McLeod. Your response should be focused on Burner’s work.
   4.2 If possible, share your discussion and findings with the other groups during the feedback session.

Section 1 focused on the discipline of mathematics by looking at mathematics as a human activity, as well as theoretical perspectives on how mathematics is learnt. Theorists such as Piaget, Vygotsky and Bruner were focused on and how their ideologies (beliefs) in Constructivism can be understood in mathematics education.

These theories are vital to consider for mathematics to be seen as a human activity and social construct. Implementing mathematics instruction based on these theories may enable learners to gain the required knowledge and skills to understand and excel in mathematics. However, some challenges are faced in South Africa that impede (hinder) this to happen. In Section 2, we consider these challenges that are similar between international countries and South Africa when teaching mathematics.
1. INTRODUCTION

There is an increase in research on primary mathematics education. In South Africa, a small number of learners continue to make noteworthy progress in mathematics; however, the majority fails to perform at appropriate levels. This, in turn, results in limited potential careers for learners who intend to work in the fields of science, health, engineering and teaching mathematics. This limits South Africa’s ability to be internationally competitive, as well as the ability to provide the infrastructure that is needed for the wellbeing of the majority of people (Grayson, 2010).

This section will discuss factors that contribute to poor mathematics performance by learners, from both an international and a national perspective.

2. FACTORS THAT INFLUENCE MATHEMATICAL PROFICIENCY

It is evident from research conducted by the Department of Basic Education, universities and other research agencies in South Africa, that the teaching and learning of mathematics need attention. Learners in South Africa perform very poorly in mathematics compared to other countries (Fleisch, 2008). This is shown in national assessments such as the Annual National Assessments (ANA), regional assessments like the Southern and Eastern Consortium for Monitoring Education Quality (SACMEQ), as well as international assessments such as the Trends in Mathematics and Science Study (TIMSS).

TIMSS has shown that South Africa’s mathematical performance is very low compared to other participating countries (Mullis, Martin, Gonzalez & Chrostowski, 2004). South Africa, however, shares similar factors that influence learners’ achievement in mathematics. Some of these factors include language diversity, diverse socio-economic conditions, overcrowded classrooms, limited resources, shortage of qualified and experienced mathematics teachers, and inadequate practices and teaching methods (Adeogun & Osifila, 2008). These factors have an impact on how learners learn mathematics and consequently their mathematical proficiency.

First, let’s look at some of the factors that influence primary school learners’ mathematical proficiency.
2.1 Socio-economic status

Socio-economic status (SES) includes both the social and economic status of an individual in a group. Poverty, rurality, ethnicity, gender, language, culture, race, among others, have been defined as the variables that constitute socio-economic influences on mathematical achievement (Valero, Graven, Jurdak, Martin, Meaney & Penteado, 2015).

Learners from higher SES receive better opportunities for intellectual, physical and emotional developments. This is because many factors such as parental involvement, family income, possession of books, etc. influence their academic performance. In many countries, there is a difference between SES and how it influences the quality of education in schools. The socio-economic conditions of different areas in India, the United States, Australia and Canada are diverse, just like in various provinces in South Africa (Valero et al., 2015).

South Africa’s history of apartheid and its resultant high levels of poverty and extreme social and economic difference between rich and poor continue to manifest in the education of learners. Valero et al. (2015) explain that South African learners’ mathematics results are below other African neighbour countries with much less wealth. Therefore, it is important for mathematics education to create new possibilities for learners. Creating possibilities for learners will help them to recognise that they are capable of doing mathematics and succeed in the future. Mathematics teachers thus should have adequate mathematical content knowledge and know-how to teach the subject to learners for them to receive quality mathematics instruction to become proficient in mathematics.

2.2 Overcrowded classrooms

Overcrowded classrooms are classrooms with a large number of learners. The teacher-learner ratio in schools is high and there is inadequate space for learners. The maximum recommended class size in South African primary school classrooms is between 35 and 40 (Education.gov.za, 2013). There are, however, many schools in South Africa that have more learners in one classroom.

Large class sizes can be overwhelming for teachers and may limit adequate instruction of mathematics. In Section 1 of this unit, you have read about Vygotsky’s and his socio-cultural constructivist theory. Taking his theory into consideration, and the importance of teachers to mediate learners’ learning during a mathematics lesson, this is a challenging task when faced with over-crowded classrooms. Teachers struggle to practice a variety of methods, such as higher-order questioning and active learning approaches in mathematics education.
Overcrowded classroom is a global challenge. Research shows that countries such as Zimbabwe, Ghana, and Nigeria have severely overcrowded classrooms (Marais, 2016). In Kenya, overcrowded classrooms can range up to 200 learners. This large number of learners have a negative impact on quality teaching and learning (Marais, 2016).

It is important that mathematics teachers find ways (strategies) to overcome this challenge to ensure quality teaching and learning in their class. As learning takes place in a social environment, social interaction between learners is important for learners to construct knowledge and learn from each other. Learners should be encouraged to find answers and solving problems together.

Read the following extract from Marais (2016) titled “We can’t believe what we see: Overcrowded classrooms through the eyes of student teachers” in the core reader and answer the questions that follow.

**Extract:**

“The influence of large numbers on learner performance

Chingos (2013) is convinced that students will learn more in smaller classes. There are more opportunities to receive individualised instruction from the classroom teacher, and therefore, parents prefer smaller classes. Parents believe that their children will perform much better in classes that do not have a large number of learners.

When comparing the achievements of large numbers of learners in classes with the achievements of small numbers of learners in classes, research conducted by Cortes, Moussa and Weinstein (2012, p. 25) highlighted the fact that class size affects student performance, due to misbehaviour and other disciplinary problems in large classes. The learners in the small classes scored, in general, much higher marks than those in the large classes. According to these researchers, learners in smaller classes learnt more as they did not experience disruption during lessons. Participation in learning activities and group work in the smaller classes contributed towards the higher scores, and resulted in sound discipline, whereas in the larger classes, learners scored lower marks, and disruptive behaviour made it difficult for teachers to manage these classrooms. From their findings, it appeared as if learners in one classroom increased the incidences of disruptive behaviour. If one or more students behave badly, the learning activities of all the learners in the class are influenced.”

1. Explain how overcrowded classrooms affect learners’ performance in mathematics.
2. Discuss the advantages and disadvantages of class size and its effects on teaching and learning.

3. In a paragraph, explain what you as the Intermediate Phase mathematics teacher can do to provide learners in an over-crowded class with ample opportunities to practice (exercise) the understanding of fractions. In your explanation provide practical examples that you can do to support learners.

Take your response to the next student academic support session and share your work with a peer or do this in your own study group.

2.3 Language diversity

Language is crucial for mathematics as it provides effective communication of mathematical ideas. McLean (2000) states that many learners’ problems in mathematics originate from inadequate knowledge of the basic vocabulary. Mastering mathematics is considered a two-step process: firstly learners need to understand the mathematical concepts, and secondly, they should be able to communicate their understanding of these concepts (Botes & Mji, 2010).

It is challenging for teachers to teach mathematics in multilingual classrooms where the medium of instruction (Language of Learning and Teaching (LoLT)) is different from the learners’ or the teachers’ language at home. In most South African schools the LoLT in the Intermediate Phase is English, which in many instances is not the learner’s language at home. This language barrier increases poor performance amongst learners. Learners are challenged with understanding the language (English) and then applying newly learnt concepts to mathematics.

Think back to your school days and write a reflection of the following:

1. What language was used to teach you mathematics?
2. How did the language of teaching and learning impact upon your understanding of mathematics.

2.4 Teacher’s mathematical knowledge

Many studies report that what teachers know (content knowledge) and believe (perceive) about mathematics is directly associated with their instructional choices and procedures (Saritas & Akdemir, 2009). Many teachers in South Africa have limited mathematical knowledge, and this has a major impact on learners’ mathematics performance (Pournara, Hodgen, Adler & Pillay, 2015).
Insufficient mathematical knowledge might impede the successful acquisition of mathematical, knowledge and skills in learners. This will influence application of mathematical skills in computations, conceptual understanding, and procedural knowledge. Therefore, to improve learner attainment and understanding of mathematics, teachers’ mathematical knowledge must be improved (Pournara et al., 2015). Teachers must be trained with the knowledge and understanding of all the relevant and appropriate mathematical concepts about the specific phase.

Sound content knowledge and understanding is an essential requirement for all mathematics teachers. The reason being if a teacher lacks the knowledge and understanding of a mathematical concept, the teacher will not teach the lesson with confidence (Ball, Thames & Phelps, 2008), e.g. Teacher X did not understand the section division of fractions. When Teacher X had to teach this section he conveniently avoids it or skim through the section. Sometimes the teacher may focus on basic level activities which may not be grade/phase appropriate. You cannot teach what you do not know! This is one of the reasons why teachers dislike teaching mathematics because they struggle with some of the content areas during their learning/training.

To be the most effective mathematics teacher it is imperative to have sound content and pedagogical knowledge. Teachers’ content knowledge should not be limited to only what should be taught but should go beyond that. If possible – far beyond! Furthermore, Ball et al. (2008) recommend that teacher’s mathematical knowledge should include an awareness of how mathematical topics and ideas are related within the mathematics curriculum for different phases. Being aware of what is expected across the curriculum will enable teachers to set the mathematical foundation required for mathematics learning that has to take place in higher grades. However, content knowledge is not the only requirement to be an effective mathematics teacher.

Ball et al. (2008) argue that pedagogical knowledge and knowledge of the learner is as important. Pedagogical knowledge will equip teachers with different teaching strategies and methods on how to teach particular mathematics content, whilst knowledge of the learner will have an impact on the teaching strategies and the content selected for teaching a particular lesson.

We will endeavour to assist you in becoming a confident and competent mathematics teacher by building and strengthening your own content knowledge of mathematics while guiding you on HOW to teach the content to learners (pedagogical knowledge). This will be done through various activities, discussions, assignments, etc.

Teachers with good content and pedagogical knowledge:
• Know how to select the most important (basic) content their learners need to know and can do;
• Can identify mathematical goals for their lessons;
• Can develop their learners’ conceptual understanding because they can teach the same concept in a variety of ways;
• Encourage learners to ask questions and make comments (because they are confident in their ability to deal with them);
• Are better in analysing learners’ mistakes, difficulties and misunderstandings;
• Are better at connecting mathematics to their learners’ daily lives;
• Are better at integrating mathematics across other subjects; and
• Can pre-empt possible misconceptions from learners.

Learners’ understanding of mathematics, their ability to use mathematics to solve problems, their confidence in and their attitude towards mathematics are all shaped by the teaching and learning they encounter in the mathematics classroom. It is, therefore, important that you keep your own content knowledge up to date and become proficient in mathematics at a deeper level than your learners. This will enable you to teach mathematics with confidence, thus improving your learners’ performance in mathematics. There are many ways you can improve your own subject content knowledge – you need to read widely, conduct research, practise solving problems (e.g. routine and non-routine problems) yourself as you proceed with your studies. In doing so, you will become skilled and your confidence will improve to teach mathematics to the Intermediate Phase learner.

120 minutes

In approximately 200 words, explain the significance of developing your own mathematical knowledge to enhance your mathematics teaching: Your explanation should make particular reference to:

• The different types of knowledge you should have and the purpose of having that particular knowledge.
• How good mathematics content knowledge will enable you to be a better mathematics teacher. Do not only list the points but elaborate, by giving practical examples.
• How your own attitude and confidence towards mathematics will impact on your learners’ performance.

Take your response to the next student academic support session or own study group meeting and share your work with a peer.

To review what you have learnt in this unit, do the next activity.
1. In a paragraph, explain why mathematics is seen as a human activity.

2. Discuss your role as an Intermediate Phase teacher in developing learners’ mathematical proficiency.

3. Read the statement and answer the question:

   “Some teachers interpret the constructivist theory of learning as meaning that knowing the content is unimportant. They argue that the teacher is a facilitator of the learners’ Mathematical explorations who can learn alongside learners” (Schuck, 1999 as cited in Luneta, 2013, p. 55).

   In approximately 160 words, explain why you agree/disagree with this statement. In your explanation make particular reference to:
   - The impact this view can have on mathematics teaching; and
   - A teacher’s content knowledge and proper planning.

4. Refer to Vygotsky’s sociocultural theory and explain how you would mediate (intervene) and support learners when teaching 2-D shapes.

5. Complete the following table to demonstrate your understanding of Piaget, Vygotsky and Bruner’s theories.

<table>
<thead>
<tr>
<th>Theorist</th>
<th>Main concepts</th>
<th>Description of theory</th>
<th>Practical example in a mathematics classroom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piaget</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vygotsky</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bruner</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Name and explain three challenges teachers may experience when teaching mathematics.

7. Refer to your response in question 6 and describe what you as a mathematics teacher will do to overcome those challenges in your classroom.

8. In two paragraphs, explain why it is important for mathematics teachers to have adequate and appropriate knowledge of mathematics. Provide examples in your response.
In Section 2 we discussed factors that South Africa and other countries face in teaching mathematics. The importance of a teacher’s mathematics knowledge was emphasised. We also discussed what is expected of a mathematics teacher to improve learner mathematical understanding and performance. This will be discussed further in Unit 2 of what learners are expected to know and how you can plan for their learning.

Before you continue, reflect on what you have learnt so far by completing the self-assessment activity. If your answer is UNSURE or NO on any of the criteria, go back to the relevant section to study it again.

### Self-assessment activity: Unit 1

<table>
<thead>
<tr>
<th>Now that I have worked through this unit, I can:</th>
<th>YES</th>
<th>UNSURE</th>
<th>NO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Define mathematics.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Describe mathematics as a social construct.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Explain different theoretical perspectives on how children learn mathematics.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Explain the factors that influence mathematical proficiency.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Identify and discuss international and national challenges to become mathematical proficient.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

You have reached the end of Unit 1. In this unit, you had the opportunity to think generally about the discipline of mathematics. You were also introduced to challenging factors that teachers may face when teaching mathematics and influence learners’ mathematical performance.

The next unit in this module will provide you with an overview of the mathematical content for the Intermediate Phase and Grade 7. Unit 2 will also focus on how you, as an Intermediate Phase teacher, need to plan for mathematics teaching and assessment, as well as provide strategies to maximise mathematical learning.
UNIT 2: MATHEMATICS TEACHING: OVERVIEW AND PLANNING

1. INTRODUCTION

Unit 1 of this module provided you with an overview of what mathematics is as a discipline. This included conceptualising mathematics as a social construct, or human activity, as well as referring to theoretical perspectives on how mathematics is learnt. Unit 1 also emphasised the need for quality mathematics education in South Africa, and that teachers should be aware of the factors that influence learners’ performance in mathematics.

Unit 2 of this module consists of three sections to provide you with an introduction to what should be taught in Mathematics in the Intermediate Phase and Grade 7. Section 1 provides an overview of the aims and skills needed in mathematics you will also look at mathematics as a subject regarding the five content areas in mathematics. The five content areas will be covered in detail in the four modules of this course over the next four years.

In Section 2, you will be introduced to different levels of planning. This knowledge will assist you in developing your own lesson plan for your class. To plan your lessons effectively, you must familiarise yourself and have a good understanding of the Intermediate Phase (Grades 4 to 6) and Grade 7 mathematics policy document (CAPS) According to the South African Curriculum and Assessment Policy Statement (CAPS) for the mathematics (DBE, 2011), the curriculum gives expression to the knowledge, skills and values worth learning in South African schools.

“This curriculum aims to ensure that children acquire and apply knowledge and skills in ways that are meaningful to their own lives. In this regard, the curriculum promotes knowledge in local contexts, while being sensitive to global imperatives” (DBE, 2011, p. 4).

As a teacher of mathematics, you need to know how learners are learning regarding research-based theories that were discussed in Unit 1. This knowledge and understanding of the curriculum and theories must guide your planning and empower you to teach effectively.

Section 3 deals with strategies to maximise mathematics teaching and learning. These include, for example, HOW to teach according to the developmental level of learners; HOW to actively involve learners; HOW to teach from the concrete to the abstract; and HOW to apply peer-assisted learning.
Section 4 of this unit focuses on **assessment** in mathematics. This will provide you with knowledge of different types of assessment used in mathematics. This section will also give you strategies on how you can assess learners in mathematics.

Unit 2 will equip you to confidently plan and teach Mathematics to Intermediate Phase (Grades 4 to 6) and Grade 7 learners.

## 2. STRUCTURE AND LEARNING OUTCOMES OF UNIT 2

Unit 2 provides you with the opportunity to work towards competency in the areas listed below.

### UNIT 2
**MATHEMATICS TEACHING: OVERVIEW AND PLANNING**

### SECTION 1
**Overview of mathematics**

**Learning outcomes:** At the end of this section, you should be able to:
- Define the aims of school mathematics.
- Describe essential mathematical skills embedded in the mathematics curriculum.
- Outline the mathematics content areas and the time allocation for each area.

### SECTION 2
**Planning for mathematics teaching**

**Learning outcomes:** At the end of this section, you should be able to:
- Use the curriculum to identify the mathematical content that must be taught in the Intermediate Phase and Grade 7.
- Explain how to plan for mathematics teaching and learning.

### SECTION 3
**Maximising mathematics learning**

**Learning outcomes:** At the end of this section, you should be able to:
- Explain how to create an inviting mathematics classroom.
- Describe strategies to teach the mathematics content effectively and maximize learning in the classroom.

### SECTION 4
**Assessment in mathematics**

**Learning outcomes:** At the end of this section, you should be able to:
- Plan for mathematics assessment in your classroom.
- Implement different types of assessment in mathematics.
1. INTRODUCTION

In Unit 1, it was emphasised that mathematics is viewed as a human activity, and reference was made to three theorists on how mathematics is learnt. You were also informed of the factors that many South African schools, and other countries, face when teaching mathematics. The importance of teachers having adequate mathematical knowledge was discussed. These are important factors/issues to consider when planning for mathematics in the Intermediate Phase and Grade 7 classrooms.

This section focuses on the Curriculum and Assessment Policy Statement (CAPS) to provide you with an overview of what is expected of an Intermediate Phase and Grade 7 mathematics teacher. The aims of mathematics teaching, fundamental mathematical skills and the content areas that should be covered in the mathematics curriculum will be discussed in this section.

2. AIMS OF MATHEMATICS TEACHING

2.1 Specific aims of mathematics teaching

To understand the world that we live in, learners need to be numerate. They need to know how to perform operations, solve problems, and most importantly, they need to know that mathematics is flexible and can be applied in many meaningful ways.

According to the Curriculum and Assessment Policy Statement (CAPS) in the National Curriculum Statement (DBE, 2011), mathematics is described as a subject that uses language, symbols and diagrams to describe numerical, geometric, and graphical relationships. Mathematics helps learners to develop logical and critical thinking skills as well as problem-solving skills that contribute towards their ability to be effective in decision making. For learners to do this, CAPS set out specific aims that teachers need to develop in learners when teaching mathematics (DBE, 2011, p. 8). The aims are listed below.

- A critical awareness of how mathematical relationships are used in social, environmental, cultural and economic relations.
- Confidence and competence to deal with any mathematical situation without being hindered by a fear of mathematics.
2.1 Aims of teaching mathematics

- A spirit of curiosity and a love for mathematics.
- An appreciation for the beauty and elegance of mathematics.
- Recognition that mathematics is a creative part of human activity.
- Deep conceptual understanding to make sense of mathematics.
- Acquisition of specific knowledge and skills necessary for:
  - The application of mathematics to physical, social and mathematical problems.
  - The study of related subject matter (in other words other subjects).
  - Further study in mathematics.

Together with these aims, the CAPS also places emphasis on fundamental mathematical skills that learners must develop while they learn mathematics.

2.2 Fundamental mathematical skills

The CAPS document (DBE, 2011, pp. 8-9) specifies six fundamental mathematical skills that learners need to develop. These skills are:

- Develop the correct use of the language of mathematics;
- Develop number vocabulary, number concept and calculation and application skills;
- Learn to listen, communicate, think, reason logically and apply the mathematical knowledge gained;
- Learn to investigate, analyse, represent and interpret information;
- Learn to pose and solve problems; and
- Build an awareness of the important role that mathematics plays in real-life situations including the personal development of the learner.

Four of these skills are discussed below.

2.2.1 Correct use of mathematical language

Learners must learn to use the correct mathematical language. This means that you, as the teacher, must use the correct mathematical language in class. We will address this as we proceed through this module. Mathematical language is made up of some specialised mathematical vocabulary used together with everyday vocabulary. As a teacher of mathematics in the Intermediate Phase, you will need to guide and develop in your learners the correct and appropriate mathematical language. You can do this by modelling good use of mathematical language yourself and by encouraging as much verbal interaction as possible in the mathematics lessons that you plan and teach.
2.2.2 Reason mathematically

Learners must be supported to listen, communicate logically and use concrete materials, diagrams and pictures to explain and describe their mathematical ideas. Doing this will develop their reasoning skills and eventually enable them to reason mathematically. Learners should also be provided with ample opportunities to experience and practice mathematical concepts in different contexts. Allowing learners to apply their knowledge and skills in various situations enables them to reason and think critically.

2.2.3 Develop number concept and calculation (computational) skills

A learners’ number concept involves the development of an understanding of what different numbers mean and how they relate to each other. Learners’ number concept is not developed in a single day or a few periods of teaching. It develops gradually over some time. Teachers must constantly work on learners’ number concept development. Their calculation (computational) skills must also be constantly improved. For example, they must know HOW to add, subtract, multiply and divide. These basic operations should be practiced as often as possible so that they can become proficient in mathematics. In using calculation methods, it is also important for learners to know how to apply a certain method, and they also need to understand why the method works or does not.

2.2.4 Apply mathematical skills to solve problems

Learners must know HOW to apply calculation methods and skills (like addition, subtraction, multiplication and division) correctly. They must also know WHEN and WHY they need to use certain methods to solve problem sums in different contexts (problem-solving situations). They also need to learn to pose relevant questions and learn different problem-solving strategies so that they can choose the best strategy to solve mathematical and real-life problems.

When you, as a mathematics teacher introduce learners to new content, you must explain every step. In this module, we focus on ensuring that learners are taken step-by-step to master the basic concepts before they learn more advanced concepts and use their acquired knowledge and skills to solve problems.
Prepare yourself thoroughly so that you can actively participate in the group discussion during the next student academic support session or in your own study group.

Based on the definition of mathematics in the introduction, the aims of the mathematics curriculum, and the skills defined above, discuss and answer the following questions in your groups:

- What are your perceptions about mathematics teaching and learning in the Intermediate Phase?
- Considering the fundamental mathematics skills given above, do you agree that these are fundamental skills? Give reasons for your answer.
- Based on the aims of mathematics and the skills learners have to be proficient in, what type of a mathematics teacher is desired by the curriculum?

Commentary:
The given questions do not have correct or wrong answers but are posed to stimulate you to start thinking about mathematics teaching and learning and the kind of mathematics teacher that is required to effectively teach the mathematics curriculum to young learners. When you discuss your perceptions about mathematics, reflect on your own experience as a mathematics learner as well as the observations you have made during your first Work Integrated Learning (WIL) period if you have already completed this.

In your response to the second and third questions, consider the aspects of the mathematics curriculum that a mathematics teacher needs to focus on. You should include aspects that have to do with language and its effective use to communicate mathematical ideas; planning and presenting mathematical ideas; providing opportunities where learners can reason mathematically, share ideas, justify their ideas, strategies and solutions to mathematical tasks; providing an inviting classroom environment; creating opportunities for learners to provide evidence of what they can do and of the mathematical understanding they have developed; and teacher feedback that guides subsequent learning.

3. THE MATHEMATICS CONTENT AREAS

Mathematics in the Intermediate Phase covers five content areas: Numbers, Operations and Relationships; Patterns, Functions and Algebra; Space and Shape (Geometry); Measurement and Data Handling.
To give you an overview of mathematics in the Intermediate Phase, let us briefly look at each of the mathematics content areas.

3.1 Numbers, Operations and Relationships

Numbers, Operations and Relationships is a key element in mathematics and focus on developing learners’ number sense. This includes the development of (DBE, 2011, p. 10):

- The meaning of different kinds of numbers;
- Relationship between different kinds of numbers;
- The relative size of different numbers;
- Representation of numbers in various ways;
- The effect of operations with numbers; and
- The ability to estimate and check solutions.

In the Intermediate Phase, attention must be focused on understanding the concept of place value so that the learner develops a sense of large numbers and decimal fractions. At least 50% of the teaching time for mathematics must be spent on this content area in the Intermediate Phase. In Grade 7, 30% of the mathematics time must be spent on Numbers Operations and Relationships.

3.2 Patterns, Functions and Algebra

Patterns, Functions and Algebra explore ideas in algebraic (numerical) thinking, such as finding, describing and using patterns and functions to make predictions (DBE, 2011). These content areas help learners to develop their ability to recognise, describe and represent patterns and relationships as well as solve problems using algebraic (numerical) language and skills.

In the Intermediate Phase, learners will work with number patterns as well as geometric patterns. Learners must use physical objects, drawings, and symbolic forms to copy, extend, describe and create patterns. Focussing on the logic of patterns lays the basis for developing algebraic thinking skills.

A central part of this content area is for the learner to achieve efficient skills in the use of numeric and geometric patterns which focus on investigation and description of patterns and relationships using:

- Words;
- Flow diagrams; and
- Number sentences.
This phase has a particular focus on the use of different, yet equivalent, representations to describe problems or relationships using flow diagrams, tables, number sentences or verbally. 10% of mathematics teaching time must be spent on this content area in the Intermediate Phase and 25% in Grade 7.

3.3 Space and shape (Geometry)

The study of space and shape is called geometry. The focus of this content area is on geometric shapes in and around the environment and improves understanding and appreciation of the pattern, precision, and beauty of mathematics in natural and cultural forms.

This content area focuses on two-dimensional shapes (2-D) and three-dimensional (3-D) objects regarding their:

- Properties and relationships;
- Orientations and positions;
- Transformations; and
- Angles.

These content areas help learners develop the ability to recognise, describe and represent characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations and positions. 15% of mathematics teaching time must be spent on Space and Shape in the Intermediate Phase and 25% in Grade 7.

3.4 Measurement

Measurement focuses on the selection and use of appropriate units and instruments. This enables the learner to measure, make sensible estimates of measurement and be alert to the reasonableness and precision of measurement results (DBE, 2011).

Measurement in the Intermediate Phase must enable the learner to measure and solve measurement problems, involving measurement of:

- Length;
- Mass;
- Capacity/Volume;
- Perimeter and Area; and
- Time and Temperature.
This content area will help learners develop the ability to use appropriate measuring units and formulae in a variety of contexts. They should be able to estimate and verify results through accurate measurement. 15% of mathematics teaching time must be spent on Measurement in the Intermediate Phase and 10% in Grade 7.

3.5 Data Handling

Data Handling involves asking questions and finding answers to describe events in the environment (DBE, 2011).

Through the study of Data Handling, the learner develops the skills to:

- Collect and organise (sort) data;
- Represent data;
- Analyse and interpret data; and
- Report on data.

This content area help learners develop the ability to collect, summarise, display and critically analyse data in order to draw conclusions and make predictions, and interpretation of information. Learners should be exposed to probability. 10% of mathematics teaching time must be spent on Data Handling in both the Intermediate Phase and Grade 7.

1. Reflect on the mathematics content areas as explained above. Which content area would you say is most important for the Intermediate Phase learner? Provide reasons for your answer.

2. On an A4 page, draw a mind map to show the knowledge and skills you will need to support learners to learn the basics of each of the five mathematical content areas.

Commentary:
The quality of mathematics teaching in your classroom will be informed by your sound understanding of each topic and how these topics relate to each other and your knowledge of teaching and learners. The selection, pacing, and sequencing of these topics require you to have specialised mathematical knowledge and knowledge of your learners.
4. TIME ALLOCATION FOR TEACHING MATHEMATICS

The time allocation per week for teaching mathematics is as follows:

<table>
<thead>
<tr>
<th>Grade 4</th>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 hours</td>
<td>6 hours</td>
<td>6 hours</td>
<td>4.5 hours</td>
</tr>
</tbody>
</table>

From the table, it is quite clear that a strong foundation for mathematics must be laid in the Intermediate Phase (see Grades 4, 5 and 6) as the time for mathematics teaching is significantly less in the Senior Phase (see Grade 7).

11 minutes

1. Consider the different strands/content areas that you need to teach in mathematics every week. Convert the allocated percentages into actual time per week for each strand. Redraw the following table to show your response:

<table>
<thead>
<tr>
<th>Strand</th>
<th>Grade 4</th>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Time</td>
<td>% Time</td>
<td>% Time</td>
<td>% Time</td>
<td></td>
</tr>
<tr>
<td>Numbers, operations and relations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Patterns, Functions and Algebra</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Space and Shape</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data Handling</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What are the implications of this time allocation for your daily planning and practice in the classroom?

3. Carefully read through the aims of mathematics teaching, the essential skills learners have to develop and the five content areas. Write an essay of 150 to 200 words to explain how achieving these will prepare/not prepare learners for the challenges they might experience in real life.

- Include suitable examples in your explanation.
- Refer to practical real-life situations to substantiate your arguments.

Note that the curriculum does not recommend an exact division of time amongst all five content areas per week but rather an estimated time of 3 hours per week to be spent on Numbers, Operations and Relationships. The rest of the time i.e. $\frac{1}{2}$ to 1 hour
per week should be spent on the other content areas. The curriculum suggests the number of periods teachers must spend on the different content areas per term.

Write down 5 aspects about mathematics and the curriculum you learnt about in Section 1 of this unit. Compare your responses to those of a peer at the next student academic support session or in your own study group.


The Grade 7 mathematics curriculum is part of Grades 7 to 9 (Senior Phase) curriculum and you can download it at: https://www.education.gov.za/Portals/0/CD/National%20Curriculum%20Statements%20and%20Vocational/CAPS%20SP%20%20MATHEMATICS%20GR%207-9.pdf?ver=2015-01-27-160141-373

These two curriculum documents are important resources that you, as a mathematics teacher, need to have available when planning and preparing for teaching.

Section 1 focused on providing you, as the Intermediate Phase teacher with an overview of the mathematics curriculum. This overview gave you an insight into what mathematics is and the specific aims of mathematics teaching and learning as set out in the curriculum. The essential skills needed for learners to acquire in the mathematics classroom were also discussed, as well as the outline of the content areas and time allocation for each. What you have learnt in Section 1, will assist you in the next section to plan for mathematics teaching.

SECTION 2: PLANNING FOR MATHEMATICS TEACHING

1. INTRODUCTION

“Planning can profitably be seen as a detailed form of instructional design aimed at reducing the uncertainties in one’s practice, centred on continual improvement of instruction and informed by scrutiny of what happens as the lesson unfolds” (Kilpatrick, Swafford & Findell, 2001, pp. 337-338).
From this statement, it is clear that you have to plan in detail to be confident in what and how you have to teach. Furthermore, in-depth reflection during and after presenting your lesson will enable you to identify challenges that should be addressed to improve your teaching.

This section aims to equip you to plan effectively by introducing you to different levels of planning for mathematics teaching. You must know WHAT you need to teach in each grade to cover the entire curriculum, therefore planning for mathematics teaching starts with the year plan. Before you start planning for your own teaching you need a clear understanding of WHY teachers plan and HOW each planning stage follows.

1. Do you plan for things in your life? Say why or why not.
2. Did you plan to study for this degree? What processes did you set in motion to make this become a reality?
3. Do you think you will succeed in your degree without a study plan? Why do you say so?
4. Now, think about why teachers have to plan? Motivate your answer.
5. What do you think mathematics teachers have to plan for their class(es)? Write down your own notes.

Commentary:
In the same way, we plan for aspects in our daily lives, teachers need to plan. We know the consequences of not having a plan when we want to achieve something; effective teaching is no different.

Teachers plan so that they can cope with the immediate demands of the curriculum as well as the needs of learners. Planning enhances teachers' confidence in the delivery of the curriculum. It helps them to cope with the unexpected and to reduce time wastage. Planning enhances the teacher’s ability to effectively manage learning and teaching in the classroom. Teachers have to plan for every lesson they teach.

The planning process will now be discussed.

2. OVERVIEW OF THE PLANNING PROCESS

Before we look at each of the planning stages, you must understand HOW each planning stage is developed from the previous stage. Study Figure 6 below which displays the different planning stages.
From Figure 6 we can see HOW the term plan is derived from the year plan. There are four school terms in a year. In each term, there are more or less 10 weeks of teaching time. Therefore, the term plan is followed by a weekly plan. As there are 5 teaching days in a week, planning culminates in a lesson plan for each day. That means that a teacher will have up to 50 lesson plans per grade per term.

We will now briefly discuss each planning stage before we get to the lesson plan which will be focussed on in detail. But first, you need to know which subject content to base your planning on.

### 2.1 The specification of content

The mathematics curricula for the Intermediate and Senior Phase prescribes the subject content for each grade and forms the foundation (basis) for planning. The specification of content (DBE, 2011, p. 12) contains the WHAT of teaching mathematics and is contained in the mathematics curriculum in the form of an overview for the three grades in the Intermediate Phase (Grades 4, 5 and 6). The same applies to Grade 7.

As explained in Section 1, mathematics contains the following content areas for each grade:

- Numbers, Operations and Relationships;
- Patterns, Functions and Algebra;
• Space and Shape;
• Measurement; and
• Data Handling.

These content areas are then distributed over the year and culminate into the year plan.

2.2 The year plan and term plan

A year plan gives you an overview of WHAT (subject content) you must teach in a specific grade for the whole year and is divided into the four terms. In other words, the year plan includes the term plans. Thus, term planning is done for you and is included in the curriculum.

The idea behind common term plans is that all schools must cover the same mathematics content in each term and each grade; that if a learner transfers from one school to another or from one province to another, this learner does not suffer content backlog. This common curriculum content will also equip learners for any provincial or national tests (such as the Annual National Assessment (ANA)) they may be required to write.

We can extract the year plan for each grade from the curriculum. Table 2 is the year plan with the time allocation per topic for Grade 4 mathematics (DBE, 2011, p. 34).
<table>
<thead>
<tr>
<th>Topic</th>
<th>Term 1</th>
<th>Term 2</th>
<th>Term 3</th>
<th>Term 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mental mathematics (10 min daily)</td>
<td>8</td>
<td>7</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>Whole numbers: counting, ordering, comparing, representing and place value (3-digit numbers)</td>
<td>2</td>
<td>1</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Number sentences</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Whole numbers: addition and subtraction (4-digit numbers)</td>
<td>8</td>
<td>6</td>
<td>1</td>
<td>Mass</td>
</tr>
<tr>
<td>Numeric patterns</td>
<td>4</td>
<td>7</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Whole numbers: multiplication and division (1-digit by 1-digit)</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Time</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Data Handling</td>
<td>10</td>
<td>4</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Properties of 2-D shapes</td>
<td>5</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Whole numbers: multiplication and division (2-digit by 1-digit)</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>Transformations 3</td>
</tr>
<tr>
<td>Whole numbers: division (3-digit by 1-digit numbers)</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>Geometric patterns 2</td>
</tr>
<tr>
<td>Number sentences</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Transformation</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Revision</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Examination</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
The mathematics curriculum contains term plans for each of the four terms in the year. You may use these in your classroom! Your term plans (for all four terms) make up your work schedule for the whole year for a specific grade.

Study the term plan for Grade 4 given above.

1. Redraw and complete the table to summarise the content prescribed for each topic for each term for the given topics.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Term 1</th>
<th>Term 2</th>
<th>Term 3</th>
<th>Term 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole numbers: addition and subtraction</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whole numbers: multiplication and division</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number sentences</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common fractions</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. For each topic in the table in number 1 above, write a paragraph to explain your observations about the progression of the content from term 1 to term 4.

2.3 The weekly plan

The term plan and weekly plan are closely linked. The weekly plan refines the term plan into weeks. There are approximately ten weeks in a term. The work to be done in the term must, therefore, be divided into the ten weeks of the term. Within this 10 weeks assessment must be included. The time allocation per topic given in the curriculum gives a good indication of how much work can be done in a week taking into account that Grades 4 to 6 learners have 6 hours of mathematics teaching time per week, whilst Grade 7 has 4,5 hours per week.

If you compile your weekly plan effectively from your term plan you will be able to complete the curriculum for the grade that you are teaching.

The WEEKLY PLAN indicates what must be taught in each of the weeks in the term.
The weekly plan must show the:

- Grade;
- subject;
- term;
- topic (topics/content that must be taught in each week);
- learning outcome of the lesson; and
- mental mathematics focus for each week.

Table 3 below is an example of a teacher’s weekly plan for Term 1 for Grade 4. Note that the topics from the term plan are now the objectives for the specific weeks. Mental mathematics must be done daily (according to curriculum specifications). The mental mathematics focus for the term is derived from the curriculum and must be spread over the ten weeks in the term.

**Table 3: Example of a weekly plan for Term 1 (Grade 4)**

<table>
<thead>
<tr>
<th>WEEKLY PLAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRADE: 4</td>
</tr>
<tr>
<td>WEEK</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>2 and 3</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6 and 7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>
150 minutes

Prepare yourself thoroughly so that you can actively participate in the group discussion during the next student academic support session or in your own study group. Should you not have this opportunity, choose one or two of the content areas and discuss your answers to the questions below with a fellow student or an Intermediate Phase teacher.

Carefully read through the 10 weeks plan for Term 1 for Grade 4 (Table 3). Use the information in the weekly plans to answer the following questions:

1. Are all the mathematical content areas covered in the 10 weeks? How does the distribution of the five content areas over 10 weeks compare with the recommended time allocation per content area discussed in Section 1?
2. In a paragraph of 80 words, motivate why the weekly plan is in accordance/not in accordance with the curriculum requirements.
3. List the title of the key content/concepts and vocabulary for each content area for each week, by copying and completing the table below.

<table>
<thead>
<tr>
<th>Mathematical content area</th>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Week 5</th>
<th>Week 6</th>
<th>Week 7</th>
<th>Week 8</th>
<th>Week 9</th>
<th>Week 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>E.g. Place value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(If you work in groups, each group chooses only one mathematical content area).

4. Compare the concepts/content and vocabulary listed for Week 1 to 10 for your selected content area.
   4.1 In a paragraph of approximately 150 words, describe what you notice about the increase in the level of complexity from week 1 until week 10.
   4.2 Write down the new vocabulary that needs to be introduced over 10 weeks and lists the Learning and Teaching Support Material (LTSM) that will be required to teach the content.
   4.3 In a paragraph of approximately 100 words, discuss how the weekly plans will inform your lesson plans for this period.
5. Share your findings and discussions with the rest of the class during the feedback session or discuss them in your study group and learn from each other.

The lesson plan will be discussed next.

### 2.4 The lesson plan

The lesson plan is the most important component of your planning – all the planning phases or steps culminate in daily lesson planning. The lesson plan forms the basis of good mathematics teaching; therefore the teacher plays a vital role in planning lessons. The teacher must be able to identify and formulate objectives, analyse content, plan learning opportunities, and consider teaching methods (Arend, 2010).

For each of the ten weeks in the term, there must be daily lesson plans showing the planning for teaching for each of the five days in the week. Lesson planning is a very challenging process. Shulman’s (1987) Model of Pedagogic Reasoning and Action as shown in Figure 7 below, helps teachers to carefully and systematically think about steps involved in planning and executing their daily lesson plans. This model involves five interrelated steps that the teacher should follow to plan and teach meaningful lessons. The subsequent section discusses the elements of this planning model.

![Shulman's Model of Pedagogic Reasoning and Action before lesson delivery](image_url)

**Figure 7: Shulman’s Model of Pedagogic Reasoning and Action before lesson delivery**

The **comprehension stage** requires you to develop and think about your understanding of the topic to be taught and how this topic relates to other topics. At this stage of the model, you are required to write out the most important content...
knowledge you want to cover in the lesson. Think about how you are going to sequence this knowledge and if possible, go to the library. Reading extensively about the topic and developing a concept map will help you to understand the topic better. Without this understanding, you are likely to teach concepts at a superficial level.

The **transformation stage** requires you to think about how to present knowledge. This stage of the planning cycle requires you to choose appropriate analogies, examples and representations that you will use to present the lesson. The model requires you to choose and think about the appropriate resources you can use to enhance teaching and learning. You also need to think about the learners and the content knowledge of this lesson.

Ask yourself questions such as the following:

- How is the content of this lesson linked to learners’ prior knowledge?
- What subject vocabulary/terminology would learners need?
- What examples can I use to help learners understand better?
- Which teaching and learning strategies are the most suitable for this lesson topic and the needs of the learners?

At this point in your lesson planning process, you need to:

- Decide on how best to transform and represent knowledge for the learners.
- Develop tasks that deepen learners’ understanding and consolidate their learning.
- Decide how best to arrange learners: e.g. individually, in twos or groups.

Note that all these questions and decisions should be dealt with during the planning stage.

As part of the **instruction stage** of your lesson, you have to plan what to tell learners and what to ask them to explain. At this stage of the cycle, you are expected to organise concepts and sub-concepts visually. You should use supporting resources and examples and speak clearly as you ask questions and give instructions.

In the instruction stage, you are expected to do the following:

- Give clear instructions for short, focused task/s;
- Give clear time limits;
- Practise the learning altogether with your learners;
- Allow learners to practise the learning in pairs or small groups;
- Allow learners to also practise the learning individually;
- Keep learners focused on their tasks;
Monitor learners’ understanding by checking their classwork; and
Provide guidance and support.

The evaluation stage of the cycle requires you to evaluate learning as you teach. At this stage of the cycle you are expected to:

- Ask questions to check learner understanding;
- Involve learners;
- Listen carefully to learner answers;
- Correct misunderstandings;
- Recap the main ideas;
- Provide explicit feedback;
- Mark learner activities; and
- Explain what needs to be completed as homework.

After lesson delivery, the next stage of this cyclic model is the reflection stage. This stage requires you to reflect on your teaching and think about:

- How your teaching enabled or hindered learning?
- What new understanding had been developed from teaching this lesson?
- How his new understanding will help you improve your practice?
- How you can adjust or change this lesson if you were to teach it again?

Consider how knowledge about the different stages of Shulman’s Model of Pedagogic Reasoning and Action (as discussed above) will influence your own planning and preparation of mathematics lessons. Write short notes on the influence on each stage:

- Comprehension stage.
- Transformation stage.
- Instruction stage.
- Evaluation stage.
- Reflection stage.

Commentary:
Continuously go back to the discussions of each stage as you think about your own planning and preparation of mathematics lessons.
From our weekly planning, we are going to develop daily lesson plans. Lesson planning is very important. Without proper lesson planning, learners will not be taught well. Actually, teaching and learning culminate (conclude) in the daily lesson plan.

From the **weekly plan** to the **lesson plan**

The lesson plan is focused on the teaching of one day only. The lesson plan is a detailed plan that focuses on HOW the lesson must be presented to learners. A written lesson plan serves as a **guide** to follow when you are teaching.

Putting your daily planning into writing will help you clarify many of your ideas. First, you must plan for a week (from your weekly plan). You must look at what you must do for the whole week and then fit the work into the five days of the week. In this way, you will know exactly WHAT and HOW to teach in each of the five days in the week.

Table 4 below, is an example of a weekly plan for Week 4 (refer to the example of a weekly plan in Paragraph 2.3 above) in Term 1 for Grade 4:

**Table 4: Daily planning for a weekly plan (Week 4)**

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objectives</strong></td>
<td>Multiplication of whole numbers 1-digit by 1-digit</td>
<td>Multiplication of whole numbers 1-digit by 1-digit</td>
<td>*Doubling and halving of numbers</td>
<td>Multiplication of 2 and 3-digit whole numbers by 1-digit numbers</td>
<td>Division of 2 and 3-digit whole numbers by 1-digit numbers</td>
</tr>
<tr>
<td><strong>Mental mathematics focus</strong></td>
<td>Multiplication facts of whole numbers to at least 10 × 10</td>
<td>Division facts of whole numbers to at least 100 ÷ 10</td>
<td>Multiplication facts of units by multiples of 10</td>
<td>Multiplication facts of units by multiples of 100</td>
<td>Mental speed test</td>
</tr>
</tbody>
</table>

Can you see that this planning is more detailed and indicates what must be taught during each day of the week? You are advised to do the same before you start with the actual lesson planning. (An example lesson plan for this topic is included in this section, see table 5.)

You must know which components to include in a lesson plan. The lesson plan must include at least the following:

- Grade;
- Date;
- Topic;
- Lesson objective;
• LTSM (Resources);
• Mental mathematics;
• Previous knowledge;
• Lesson presentation;
• Questions and homework; and
• Reflection

NOTE: A SANTS lesson plan template is included in Addendum A for you to plan your own lessons during Workplace Integrate Learning (WIL) and in your own teaching. It is also available on mySANTS in electronic format.

We will first explain each component and then provide you with an example of a lesson plan for mathematics. Study the lesson plan so that you can get an idea of HOW a lesson plan must look. Understanding each component will assist you to plan your own lessons thoroughly. In the modules that follow, you will develop your own lesson plans for various mathematics topics contained in the curriculum for specific grades. We will assist you to develop lesson plans for each lesson that you must teach.

Here is a brief explanation of each of the components of the lesson plan:

Grade

The grade of the learners you are teaching, for example, Grade 4.

Date

Each lesson must be delivered on a specific day. It is, therefore, necessary to indicate the date on which the lesson is to be presented.

Topic

The topic of a lesson is regarded as the content focus of your lesson. The topic is related to the objectives of the lesson as it states WHAT will be covered during the lesson, for example, Number Sentences. Please take note that the topic should be derived from the content area specified in the mathematics curriculum.

Lesson objective

The objective of the lesson is the goal that you set out to achieve. Use measurable verbs when formulating your objectives. This will help you to measure whether learners have achieved or not achieved the objective. The objective of the lesson is derived from the lesson topic. Your lesson should be built around the lesson objective(s). If you are certain about the objective of the lesson, the planning becomes much easier.
Example:
Grade 4 – Mathematics (Intermediate Phase)
Topic – Number sentences
Lesson Objective – Learners must solve simple number sentences

Having clarity on the objective of the lesson will assist you to identify the subject content for the lesson, the HOW (methodology) and the assessment activities of the learners. Then you can decide on the appropriate LTSM to use when you present the lesson.

Learning and Teaching Support Material (LTSM) for the lesson

Resources or LTSM must be chosen carefully. The function of LTSM is to enhance learning. As mentioned before, having a clear understanding of the objective of the lesson assists you in deciding which LTSM will be needed to support your teaching and enhance the learners’ learning. In the example referred to above – the objective of the lesson being: Learners must solve simple number sentences, the LTSM needed will be, for example, be a self-made balance scale. If used correctly, it can assist learners in understanding how to balance a simple number sentence to solve it.

Mental mathematics

Ten minutes per day must be spent on mental mathematics. Learners must know the basic addition and subtraction facts as well as the multiplication tables. There are specific curriculum requirements for mental mathematics for each grade. Mental mathematics will be dealt with in more detail in the second module of this course (Mathematics Teaching in the Intermediate Phase 1).

Informal class assessment and learners’ prior knowledge

You must know exactly what knowledge your learners already possess to build on that knowledge in the lesson you are going to present. No lesson is a separate entity, but each forms part of a meaningful whole. You cannot start teaching new content if your learners have not mastered the previous day’s work. You should start a lesson with a recap of what you covered in the previous lesson. You can give your learners a short class test before you start with the new lesson. If your learners had homework from the previous lesson you need to go over it with them before you start with the new lesson.
Read the discussion above under the headings Grade up to Informal class assessment and learners’ prior knowledge to answer the following questions:

1. At what stage of Shulman’s Model of Pedagogic Reasoning and Action should you think about the discussion points above?
2. How will your thinking and planning help you with your lesson delivery?

Commentary:
The sections above require you to think about the grade, time, date, topic and purpose of your lesson, teaching and learning resources you will need, activities for your whole class, oral mental mathematics and learners’ prior knowledge. Refer to the stages according to Shulman’s Model of Pedagogic Reasoning and Action and think about which stages enable you to think before you start delivering the lesson. Thinking and planning your lesson beforehand will help you to use your teaching time productively.

Lesson presentation

Lesson presentation is also referred to as a methodology in this module. Without the HOW it is not possible to present the content of the subject to your learners. HOW you are going to present the lesson is very important. When teaching mathematics, you must always move from the concrete to the abstract. In your lesson plan, you need to explain what you will do and what learners will do during the lesson, HOW you will use the LTSM to advance learning. You should also think about possible assessment strategies to assess learners’ understanding during the lesson presentation.

Questions and homework

You need to continuously assess your learners’ knowledge and skills to make sure they understand and have mastered the subject content presented to them. That means that you must ask questions during and also after the lesson. Doing this, you will be able to identify the learners in need of extra support as well as the learners that can do enrichment activities. Always invite all the learners to participate in the enrichment activities – do not only call certain learners by name. It is important that the learners are exposed to challenging, problem solving questions. Without
assessment, you will not know if your lesson was successful or not. You should give learners homework activities a few times a week, not only to reinforce what they have learnt but also to offer opportunities to them to apply their new knowledge in different contexts. Always mark the homework in the next lesson and allow learners to participate when you do this.

19

60 minutes

1. Which stage of the Pedagogic Reasoning and Action Model tells you about the assessment of learning?
2. Why is it important for you to evaluate and assess while you teach?
3. When do you have to think about the evaluation and assessment of learning in the process of lesson planning?
4. What do you think about giving homework to Intermediate Phase learners? Do you agree that learners need to get homework? Motivate your response.

Commentary:
Although the lesson must start with an assessment of prior learning so that you can build on learners’ previous knowledge, it is the evaluation stage of the cycle that tells you about the actual learning that took place in (during) the lesson. There are guidelines for various aspects of a teachers’ work relating to the assessment given at this stage of the pedagogic cycle. Ongoing assessment in a classroom is critical because it enables the teacher to track learners’ progress. Assessment is not just a marks-gathering exercise, it is something that you use to make sure that learners are learning the concepts and skills you are teaching them. It also gives insight into how to remediate identified problem areas. You need to start thinking about assessment at the outset when you start to plan because your assessment must align with your teaching programme. It is not something you would add at the end as an afterthought.

An example of a lesson plan follows next in Table 5. You will find the SANTS lesson plan template for the Intermediate Phase in Addendum A. Refer to the daily planning for Week 4 (see Table 4) to see where this lesson fits into the week. Remember to still do the daily ten minutes mental mathematics.
### Table 5: Example of a lesson plan for mathematics on SANTS' lesson planning template

**SANTS Private Higher Education Institution**  
**GRADES 4, 5, 6 and 7 LESSON PLANNING FORM**

<table>
<thead>
<tr>
<th>1. SUBJECT</th>
<th>Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. TOPIC</td>
<td>Doubling and halving numbers</td>
</tr>
<tr>
<td>3. DATE</td>
<td>2020-02-19</td>
</tr>
<tr>
<td>4. GRADE</td>
<td>5 6 7</td>
</tr>
<tr>
<td>5. CRITICAL OUTCOMES</td>
<td></td>
</tr>
<tr>
<td></td>
<td>✔ Identify and solve problems and make decisions using critical and creative thinking.</td>
</tr>
<tr>
<td></td>
<td>✔ Work effectively with others as members of a team, group, organisation and community.</td>
</tr>
<tr>
<td></td>
<td>✔ Organise and manage themselves and their activities responsibly and effectively.</td>
</tr>
<tr>
<td></td>
<td>✔ Collect, analyse, organise and critically evaluate information.</td>
</tr>
<tr>
<td></td>
<td>✔ Communicate effectively using visual, symbolic and/or language skills in various modes.</td>
</tr>
<tr>
<td></td>
<td>✔ Use science and technology effectively and critically showing responsibility towards the environment and the health of others.</td>
</tr>
<tr>
<td></td>
<td>✔ Demonstrate an understanding of the world as a set of related systems by recognising that problem-solving contexts do not exist in isolation.</td>
</tr>
<tr>
<td>6. SUMMARY OF THE CONTENT TO COVER IN THIS LESSON</td>
<td>The focus of the lesson is on the concepts 'doubling and halving' numbers. It is also important that the learners and I, the teacher, use the correct vocabulary when explaining or giving feedback.</td>
</tr>
<tr>
<td>7. LESSON OBJECTIVE(S)</td>
<td>By the end of the lesson the learners should be able to…</td>
</tr>
<tr>
<td>7.1 PRE-KNOWLEDGE</td>
<td>Double and halve numbers.</td>
</tr>
<tr>
<td></td>
<td>Give learners a class test of 5 minutes on the content of the previous lesson.</td>
</tr>
<tr>
<td>7.2 CONCEPTS and NEW KNOWLEDGE</td>
<td>Develop the correct use of the language of mathematics.</td>
</tr>
<tr>
<td></td>
<td>Develop number vocabulary, number concept, calculation and application skills.</td>
</tr>
<tr>
<td></td>
<td>Concepts and skills related to doubling ad halving of numbers.</td>
</tr>
<tr>
<td>7.3 LESSON OBJECTIVES</td>
<td></td>
</tr>
</tbody>
</table>
7.4 FUTURE LEARNING
(Briefly describe what they will learn in the lesson that follows this one)

In the following lesson, the focus will be on multiplication of 2 and 3-digit whole numbers by 1-digit numbers.

8. LTS(M (Name LTS(M you intend using in this lesson and remember to reference ALL your resources (text books, websites, workbooks etc.) under Reference list of all sources consulted in point 9 below. Try to use a number of different items or types of LTS(M.)

I need the following to teach this lesson:

• A number line from 0 to 100.
• Eight envelopes (2 of each) that contain 4, 8, 16 or 32 matches. (I must prepare this beforehand).
• Additional match sticks.

9. REFERENCE LIST OF ALL SOURCES CONSULTED
(List all the text books, workbooks, websites etc. that you used to prepare this lesson)


10. LESSON PHASES
10.1 INTRODUCTION OF THE LESSON
(Time allocated: 10 min)
(Give a detailed description of how you greet the learners, how you set the atmosphere for the new lesson, how you awaken the learners’ prior knowledge, and how you create a link between what the learners already know and the new knowledge that you will be presenting)

• After I greeted the learners and they did the mental mathematics, I will ask the learners what ‘double’ means. They must give examples and explain. I must lead learners to conclude that ‘doubling’ means to make something 2 times more in size or quantity.
• I will divide my learners into groups of 6 - 8 learners.
• Then I will provide each group with an envelope that contains some match sticks i.e. 4, 8, 16 or 32. Group leaders must collect additional matches for the group work. I will ask learners to use additional matches to double their number of matches in the envelope. They will do this practically. They must give feedback and explain their reasoning to find the answers. While facilitating the group work I will ask questions like:
  o How can you be sure that the matches are now “doubled”?  
  o Can you prove that you now have double the number of matches in the envelope? 
  o What would be the total if there were 15 matches in the envelope? 
  o What is double 50?
• Learners must now pass their envelope to the next group. I will manage the situation to ensure that each group gets a new envelope.
• Then I will ask learners what ‘halving’ means. I will lead them to conclude that ‘halving’ means to cut in half or divide by 2.
• Learners must now halve the matches inside the envelope. They must do this practically. They must give me feedback and explain their reasoning to find the answers. While facilitating the group work, I will ask questions like:
  o How can you be sure that the matches are now “halved”?  
  o Can you prove that you now have halved the number of matches in the envelope? 
  o What would you get if there were 16 matches in the envelope?
  o What is half of 100?

10.2 DEVELOPMENT – PRESENTING THE NEW KNOWLEDGE
(Time allocated: 10 min)
(Give a detailed description of WHAT content you are presenting, HOW you will be presenting it, and WHAT THE LEARNERS WILL BE DOING):

• In the next stage of the lesson, I will teach the whole class.
• I will write the numbers 2, 4, 8, 16, 32, 64 in one line on the chalkboard. Here I will ask learners to:
Show these numbers on the number line.
See if they can find a pattern.
Learners may come up with various ideas, but I will lead them to realise that each number is twice as much as the one before it. It was therefore doubled.
I will refer to the group work done earlier and assist learners to see that:
- 4 is double 2,
- 8 is double 4,
- 16 is double 8, etc.
Then next I will let learners study the pattern backwards: 64, 32, 16, 8, 4 and 2. I will ask learners to see if they can find a pattern and show the numbers again on the number line.
In doing this I will refer to the group work done earlier and assist learners to see that:
- 32 is half of 64,
- 16 is half of 32,
- 8 is half of 16, etc.
When I feel sure that my learners have grasped the concepts “double” and “half” they are ready for written work.

10.3 CONSOLIDATION
(Time allocated: 10 min)
(Give a detailed description of how you will consolidate the new knowledge, as well as how you incorporate assessment of the objectives and how you will wrap up. Please also mention here any HOMEWORK that you will give the learners.):

Let learners do written work (start in class – to be completed at home)
My learners will complete an exercise working with a partner where they double and half given numbers. They can work from the chalkboard, a worksheet or their textbooks.

It is easier to grasp the concepts if they are recorded in table form. I will, therefore, write the exercise on the chalkboard (If there is a similar exercise in your learners’ textbooks, they may use that or you can prepare a worksheet). I will give my learners 10 examples (like the 5 shown below) to do individually in their class workbooks. They must start in class and complete their work at home.

<table>
<thead>
<tr>
<th>Half</th>
<th>Number</th>
<th>Double</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>36</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>110</td>
<td></td>
<td>248</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td></td>
</tr>
</tbody>
</table>

I will remind learners to revise doubling and halving and complete their written work at home.
I will tell learners that the next day’s mental work will be based on doubling and halving.

In this lesson, I will do a continuous assessment. As learners proceed with the lesson, I will ask questions to make sure that they understand the concepts of “doubling” and “halving”.
At the end of the lesson I will ask again:
- What does it mean to double a number?
- What does it mean to halve a number?
- What is double 300?
- What is half of 600?
Learners must complete their written work on halving and doubling at home.
### 11. DIFFERENTIATION

#### 11.1 LEARNER ENRICHMENT

(Indicate what measures are in place for learners who grasped concepts quickly. How will you challenge them and keep them from getting bored?)

I realise it is important for learners to be exposed to problem solving questions. For the learners that grasped the concepts quickly, I will hand out a worksheet with riddles for them to solve: E.g.

*I am a number between 15 and 30. If you double me, my value is half of 80. What number am I?*

The learners can also create their own riddles once they have completed their homework.

#### 11.2 LEARNER SUPPORT

(Indicate what measures are in place for learners who struggle to grasp the concepts. How will you support them and keep them from getting negative and frustrated?)

There might be learners who struggle to grasp the concepts. I will walk around in the class and ask learners certain questions while they are doing the written work in order to determine if they understand. Once I have identified the learners in need of extra support on this topic, I will call them to my desk and in a group I will go through the work again, maybe slower if necessary and on a semi-concrete level (or even concrete level if needed).

### 12. REFLECTION

Briefly reflect on your lesson by discussing its strengths (what went well), its weaknesses (what did not work), what you find challenging, if the lesson objectives were met and what you would improve if you had to teach this lesson again. Use the following questions to guide your reflection:

1. Describe aspects of your lesson that worked really well.
2. Which areas of your lesson did not go according to plan? Explain why you think this may have happened.
3. Re-examine your lesson objectives. Did you meet them? Why/why not?
4. This is what I learnt about the learners in my class today.
5. What was my most challenging moment in this lesson and why? How will I respond next time?
6. To what extent were the learners productively engaged in the learning process? Discuss.
7. If I had the opportunity to teach this lesson again to this same group of learners, what would I do differently? Why?
8. What evidence/feedback do I have that the learners achieved an understanding of the lesson objective(s)?

### 3. REFLECTION

This is a very important part of the lesson that is often neglected by teachers. This part of the lesson plan must be completed **after the lesson**. It gives you an opportunity to think (reflect) about the lesson – What worked well? What needs to be changed? Did I cater for those learners who experience barriers to learning? Were there expanded opportunities? How can I improve my lesson? For example, the learner activity or selected LTSM was not effective and did not advance learners’ understanding or too much content was selected for 30 minutes. You need to make notes of your reflection so that you can use it when you present the same lesson again.
Prepare yourself thoroughly so that you can actively participate in the group discussion during the next student academic support session (or do this activity with a peer in your own study group).

In your groups, compare the above lesson plan (see Table 5) with the weekly plan for Week 4 (see Table 4) in this section, before answering the following questions:

1. Identify the concept(s) that was focussed on during this lesson.
2. List the concepts that still need to be addressed in the remaining days of week 4.
3. Write down two instructions in this lesson that can be used effectively when teaching Multiplication of whole numbers: 1 digit by 1 digit.
4. What do you think will work well if the lesson plan is implemented in class? Give reasons for your answer. What can be improved on? Also provide reasons.

Share your findings and discussion with other groups or within your own study group.

1. Write a paragraph of approximately 80 words to answer the following questions: If you find that your learners have not mastered the content adequately, do you think the teaching method used could have affected? Why/why not?

2. In approximately 80 words, discuss how a teacher’s teaching methods can be adapted to help learners to understand the new content.
   In your discussion:
   - Give an example of a teaching method that did not work well in your class (from observing your mentor teacher or from your own teaching during WIL).
   - Give an example of a teaching method that worked very well in your class. Your learners enjoyed the lesson, learnt from your teaching and were able to apply this new knowledge.
   - Make a list of different teaching methods that can be used to teach mathematics in the Intermediate Phase and Grade 7 class.
Read pages 92 to 106 of the core reader on *Improving the quality of mathematics teaching with effective planning practices* by Akyuz, Dixon and Stephan (2013), before answering the following questions:

1. On A4 paper, draw a mind map to summarise the impact of each of the following practices in mathematics teaching:
   - Practice of preparation;
   - Practice of reflection;
   - Practice of anticipation;
   - Practice of assessment; and
   - Practice of revision.

2. Compare the summarised practices with the example lesson plan in Table 5 above. Reflect on your observations by doing the following:
   - Identify and highlight the differences and similarities.
   - Explain how you will modify the lesson plan to be in line with the summarised practices. In your explanation, also refer to how the lesson plan is underpinned by the summarised practices.

Compare your response with a peer during the next student academic support session or in your own study group.

Section 2 focused on using the curriculum to identify the mathematical content that should be taught in the Intermediate Phase. This will assist you in planning for mathematics teaching and learning.

In Section 3 we consider some ideas about how to maximise learning, i.e. how to be an effective mathematics teacher in the Intermediate Phase and Grade 7.

**SECTION 3: MAXIMISING MATHEMATICS LEARNING**

1. **INTRODUCTION**

Do you want to be a successful, inspiring mathematics teacher? Then you should feel that the task you have undertaken is worthwhile. You also need to know how to create an inviting classroom and an effective learning environment to maximise mathematics
learning. It is important that learners must not only be able to do sums, they must also understand and enjoy mathematics.

Mathematics teachers themselves can do a lot to create an effective learning environment to maximise learning. Teachers, however, experience challenges, such as the factors discussed in Unit 1, Section 2 that impedes them to create an effective learning environment. Let us look at what is meant by a conducive and inviting learning environment for the mathematics classroom and what can a mathematics teacher do to create this.

2. AN INVITING MATHEMATICS CLASSROOM

Some teachers have more than 50 learners in their mathematics classes and not enough space; while others do not have their own classroom. Some do not even have resources / LTSM at the school. What can a teacher then do to improve classroom conditions?

Although you might not teach in an ideal environment, you can still find ways to enrich your classroom environment while only having the bare minimum (as a structure) in place! Doing something about appalling conditions in the classroom is not written in a teacher’s duty roster. It is not something that can be expected of teachers. It is something teachers do because they care or have a passion for teaching.

Only when teachers personalise the challenge of the existing classroom environment, will the problem be addressed. Ignoring the problem is not an option and a negative attitude towards it will not solve the problem either. Therefore, for a teacher who has more than 50 learners in her classroom and not enough space or furniture, these teachers need to think on their feet on how to make teaching and learning exciting and meaningful. First step: Change your attitude towards your situation - it is the only way in which teachers will become involved in trying to find solutions. They need to change their attitude as depicted in Figure 8 below.

![Figure 8: Changing attitudes amongst teachers](image-url)
What the teacher does with the available facilities and classroom situation is what counts. Whether your classroom is a well-built brick room or a fragile shack, improvement is possible.

When you are the teacher, you must evaluate your classroom environment by asking yourself:

- Does my classroom environment support learning?
- Which elements need improvement?
- Which of the elements that need improvement can be considered as a priority?
- How can the classroom be improved?

An attractive, well lit, comfortable and colourful classroom is the ideal situation, but that is not always possible. You, as the teacher, must see to it that within the limitations you have to face, you create a classroom environment that will benefit your learners and maximise learning.

Before we continue to discuss the physical arrangements of a classroom, what would you say is the simplest, yet effective, way to improve the classroom environment without any costs involved? Yes, it is cleanliness and neatness. To keep a classroom clean costs very little! It only requires commitment and a little effort.

2.1 Hygienic and clean classroom

Having a hygienic and tidy classroom must be prioritised as it is the first step in preparing a conducive teaching and learning environment. A teacher must plan and organise the cleaning of his classroom. For example, ask learners to sweep the floor, dust the classroom and to close the windows each day after school. The classroom must always be tidy – learners should be taught not to leave papers lying on the floor. LTSM must be neatly packed away after teaching. If learners stay in the same classroom for the day they must, under your guidance, take responsibility for the cleanliness of their classroom. In this way, they can take pride in their learning environment.

In Japan, for example, it is tradition that the learners do the cleaning of the classroom themselves as part of their many responsibilities (Menezes, 2019). Maybe we in South Africa can learn from that.
Study the pictures of the two classrooms (Classroom A and Classroom B) given next and then do the activity that follows.

Classroom A
Source: Travelationship (2010)

Classroom B
Source: Knitty (2010)

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1. Compare the two classrooms depicted in the pictures above and tabulate the differences. Make particular reference to the following in your table:

   - The physical condition of each classroom (infrastructure);
   - Availability of LTSM; and
   - Hygienic and neat classroom.

2. Which of the two classrooms would you prefer to teach in – Classroom A or B?

   - In a paragraph of approximately 100 words, explain why you prefer the selected classroom.
In your explanation make particular reference to the factors mentioned in Question 1.

3. What are some of the things you would do you change the appearance of classroom A. Make a list.

### 2.2 Furniture arrangements

Although you have no control over the size of your classroom, you can control the seating arrangement in your classroom. The seating arrangement should accommodate the learning activities. Rigid one-size-fits-all physical facilities will not be suitable for all learning activities, will not meet the needs of many learners, and may impede their learning.

To enable interaction among learners and between teacher and learners the most common arrangements are rows, clusters of two or more desks or semi-circles. The degree of interaction and teacher directedness varies with each arrangement. The degree of interaction you want between you and your learners and among your learners should be the key determinant in selecting the seating arrangement.

Two possible floor plans are given.

<table>
<thead>
<tr>
<th>Floor plan A</th>
<th>Floor plan B</th>
</tr>
</thead>
</table>

![Floor plan A](image1.png) ![Floor plan B](image2.png)

In floor plan A we have an arrangement where desks are in rows. The learners sit in straight rows and face the chalkboard. The teacher’s desk is in front of the class and s/he faces the learners. 40 learners in the class are seated two at a desk. Naudé and Meier (2015) suggest this seating arrangement is appropriate for discovery learning and that needs to be individualised. This seating arrangement is also appropriate for whole-class teaching.
In floor plan B there are also 40 learners in the class seated at 20 desks, but the desk arrangement is different. Here, the teacher’s table is in the back corner of the classroom. The block with the big circle (●) represents the teacher. She is standing next to the chalkboard. The desks are arranged in such a way that the learners face one another. This arrangement will promote interaction during group activities. Naudé and Meier (2015) recommend that, for group activities, learners should not be grouped according to their abilities as this may result in the labelling of learners. Instead, they recommend mixed ability grouping as learners who struggle to master concepts can be supported by their peers.

Classroom furniture must be arranged for productive learning. We need to consider organising the classroom furniture according to the learning activity that must take place. Learners’ chairs should face the place where most teaching will take place. You must make sure that all learners have an unrestricted view of the chalkboard. The teacher should also be able to see and monitor all learners at all times from anywhere in the classroom.

2.3 Mathematics corner

Every mathematics classroom should have a mathematics corner. In the mathematics corner, counters and other LTSM for mathematics, mathematics activities, games and supporting worksheets can be stored. Use suitable mathematics posters to create an inviting atmosphere. If you have your own classroom, use your creativity to make the best use of your classroom space. The bottom line is: your classroom must look like a mathematics classroom. If you teach mathematics in different classrooms, each of the classrooms must have its own mathematics corner as depicted in Figure 9 below:

Carefully read the statement below, before completing the task that follows.

“All learners must have an unrestricted view of the chalkboard.”

Explain why floor plan B (given above) does not meet this requirement. Copy and adjust floor plan B in your workbook to illustrate how you will change the furniture arrangement to meet this requirement.

Share your modified floor plan with a peer during the next student academic support session or in your own study group.
You can make most of the LTSM you need to use in your mathematics teaching, for example:

- Counters,
- Number lines,
- Number expansion cards (spray cards),
- 100 charts,
- Fraction walls,
- 2-D shapes,
- 3-D objects,
- Measuring apparatus,
- Money and goods,
- Clocks and timers,
- Graphs, and
- Worksheets.

Think about the (waste) material that could be available in your own environment.

1. Make a list of at least 10 free items you can collect from your environment that can be used effectively in your classroom to support mathematics teaching and learning.

2. In a paragraph of approximately 80 words, discuss how you will use each of the listed items to enhance learners’ mathematics understanding.
Take the list to your next student academic support session or your study group and exchange your ideas in your group. Write down ideas that you do not have on your list.

2.4 Classroom display

Ensure that learners can easily see wall displays (the posters, media and learner’s work posted or exhibited on the walls of the classroom). These should be displayed (shown) on learners’ eye level. When learners see their work on display they take pride in what they are doing. Make sure an area within the classroom is used for displaying learners’ own work. When teaching mathematics in different classrooms, each classroom must have its dedicated display space for mathematics. The classroom must still look like a mathematics classroom!

Sections of the walls and cupboards can be used to display important reference material, for example, a number line or fraction wall. In the picture below (Figure 10) the display focused on fractions.

![Figure 10: Display on fractions](Source: The Teacher Wife (2012))

Place charts, posters and pictures regarding the current topic/theme of learning in strategic places as shown above. Another resource that is imperative in teaching fractions and which are not displayed above is a fraction wall. Such a prominent
display of material relevant to the current topic will help learners to refer to basic knowledge when they get stuck. These base-line posters and charts will assist in teaching learners how to look for answers to questions or recap/recall of knowledge and understanding.

Each mathematics classroom in the Intermediate Phase should have a number line as well as posters, charts and pictures related to the different mathematics content areas. Other sections of the walls and cupboards might be filled with information on classroom procedures and rules. Learner’s work can also be displayed.

You may ask where you will get all the colourful posters, number lines and fraction walls we are talking about. It is also true that most schools do not have any LTSM! The solution is to make your own LTSM for mathematics teaching. You do not need to buy expensive resources, it only requires your creativity, commitment and a little (or maybe a lot) effort from your side.

2.5 Storage space

You should make storage space and materials easily accessible to learners. Here are some examples of how LTSM (resources) can be stored:

![Figure 11: Examples of how LTSM can be stored](image)

Frequently used material, e.g. counters should be stored in labelled containers or on shelves and be easily accessible to learners when they are needed as shown in the pictures above. Household items, for example, empty coffee tins, plastic cool-drink bottles, empty ice-cream containers and/or shoe boxes, can be used.
Materials and resources that are rarely or not regularly used can be stored out of the way. Teachers need to demonstrate to learners HOW to use and clean learning materials and how to store them away when not in use.

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What would you consider when creating a conducive/inviting classroom to stimulate learning and promote learner participation?

First jot down a few points and then write an essay of approximately 600 words on at least 4 key ideas.

Commentary:
Keep in mind that the arrangement of a classroom makes a big difference to the learning and teaching that will take place. Read the information given above and write down some ideas for creating a conducive learning environment in your own classroom. Think about aspects such as the neatness of the room, furniture arrangement, a mathematics corner, displays on your classroom walls and how you would store all the things you need to keep in your classroom. Do not just refer to the information supplied in the CLG, also incorporate your own ideas and practical experience.

[Revisit the Academic Literacy (B-ALI 110) module dealt with in the first semester of Year 1 to refresh your mind about essay writing.]

Now that you know HOW a learning environment that will enhance mathematics teaching and learning must look, we will explore some strategies to maximise mathematics learning.

3. STRATEGIES TO MAXIMISE LEARNING

Strategies to maximise mathematics learning include teaching according to the developmental level of learners; actively involving learners; teaching from the concrete to the abstract; and peer-assisted learning. Each of these strategies will now be discussed.

3.1 Teach according to the developmental level of your learners

You were introduced to child development and how children learn in the modules: Education Studies 1: Theories of Child Development (B-EDS 111), and Education Studies 2, Theories of Learning and Teaching, (B-EDS 122). In these modules, you
learnt about the different stages of development and how, at every stage, children progressively become more independent.

Be aware that Intermediate Phase learners in the same grade function on different developmental levels. You must therefore always consider your learners' developmental level, strengths and weaknesses when planning for teaching, learning and assessment. Reys, Lindquist, Lambdin and Smith (2014) claim that learners in a class have diverse characteristics, in other words, all learners differ. Nevertheless, there are common characteristics that are typical of all learners in a particular age group.

Important characteristics of Intermediate Phase learners are:

- They have a spontaneous urge to master things, to learn and to develop their potential. Learners WANT to discover, explore and experiment with new things. Learners in the mathematics class must be supported to develop their own abilities.
- They are by nature interested in those things that have meaning or are important to them. If you link the work to what the learner knows and experiences, learners will be interested. Your mathematics lessons must, therefore, always take into account and correlate with what learners already know.
- They are active learners who enjoy the challenges of working alone as well as with others. Keep this in mind when you plan mathematics learning activities for your learners. Vary your activities between group and individual activities.

Keep all these characteristics in mind when you plan mathematics learning activities for your learners. Here are some valuable hints to ensure that you teach at your learners’ developmental level:

- First, encourage learners to experience concepts with concrete materials, then link to semi-concrete representations before you use abstract mathematical symbols.
- Use various teaching strategies and a variety of representations (like pictures and models).
- Let learners regularly explain their thinking.
- Provide learning activities that encourage active involvement.
- Provide learners with ample opportunities to solve problems.
- Give learners constructive support and regular feedback.

To lay a strong foundation for mathematics, learners’ personal experiences and prior knowledge about a mathematics topic must also be considered. You, therefore, need to identify and build on what your learners already know about a topic. Their prior
knowledge provides a platform for you on which to actively build new mathematical knowledge. In the Mathematics Teaching modules that follow, you will find out more about how to do this.

### 3.2 Actively involve your learners

According to Reys et al. (2014), learning mathematics is not a spectator sport. Learners need to be **actively involved** in the learning process. In this manner, learners will develop a deeper understanding and what they learn will make sense to them. When they are actively involved in the learning process by doing mathematics, they construct their own mathematical meaning, i.e. they make sense of what they are doing. Active involvement requires learners to be physically active (handling learning material) and mentally active (Reys et al., 2014). Therefore, mathematics activities must provide for physical involvement through movement and the involvement of as many senses as possible while interacting with materials. Furthermore, they should provide for mental involvement through activities that require thinking, making decisions or trying out different methods rather than just methodical (rote or parrot-like) application of algorithms (recipes).

Learners must be actively involved in, for example:

- Using (handling) LTSM in class – they must measure, build, construct, etc.
- Writing in their class workbooks – they must **do** lots of mathematics!
- Completing worksheets – they must practise their skills!
- Asking and answering questions.
- Solving problems themselves.

To keep learners focused on and interested in learning activities, vary between group, pair and individual activities. Intermediate Phase learners want to do things for themselves. Thus, they should be given ample opportunity for self-activity in the mathematics class.

As active involvement also requires learners to be mentally active, they should be given sufficient opportunity to answer and ask questions to promote their thinking and decision-making skills. To be able to answer questions, learners should be encouraged to listen. Listening skills need to be developed. The teacher needs to ask questions to keep the learners’ attention focused on the lesson. To encourage learners to ask questions, they need to be in an inviting environment where they feel safe and secure. Tell learners that if they don’t know answers to questions, they can always ask the rest of the class what they think the answer is, or tell the learners you will give them an answer the following day. Learners should also feel safe to make mistakes and, with guidance, be allowed to self-correct.
In their involvement with mathematics, learners should be assisted to construct their own meaning of mathematical concepts. Learners ought to become doers of mathematics – they must be **actively involved in doing mathematics**. Learners must be involved in meaningful activities. The mathematics that they do must make sense to them. Meaningless (parrot-like) activities like rote counting (counting without understanding) must be used selectively and seldom.

In the mathematics class, learners need to explain their reasoning and justify their answers. It is the teacher’s responsibility to ensure that learners get the opportunity to do this and to constantly check learners’ answers as well as their reasoning.

To ensure that learners reach an understanding of the mathematics they are doing, you need to continuously ask them:

- WHAT are you doing?
- WHY are you doing it?
- HOW are you doing that?
- Is there another way of finding the answer?

Reys et al. (2014) suggest, among others, the following to be considered for the enhancement of effective mathematics learning:

As a teacher you have to:

- Model problem-solving approaches rather than giving solutions. The focus should be on the process rather than the product or answer.
- Encourage intellectual risk-taking and help learners feel safe about taking risks, by allowing them to do activities themselves and not to be too critical. They must also not be scared to pose questions!
- Help learners understand that confusion, partial understanding, incorrect answers, errors, and some frustration are natural as they construct their mathematical knowledge.
- Help learners understand that they will not all learn the same things at the same time and that they will not all be equally capable or skilled, but that everyone can become proficient.
- Encourage and praise learners for critical thinking and creative problem solving so they learn to value and respect those approaches. Remember that learners who are exposed to a problem-solving approach to mathematics perform better than those who only experience and focus on recipes and procedures.

Always ensure that you actively involve your learners, that your lessons are worthwhile and interesting and that your learners look forward to the next mathematics lesson.
3.3 Teach from the concrete to the abstract

Good mathematics teaching proceeds from the concrete to the abstract. This strategy of moving from the concrete through the semi-concrete to the abstract is based on theories of how children learn, notably, the theory of Jerome Bruner. You learnt about this theory in Unit 1, and you will learn more about this in the module: Education Studies 2: Theories of Learning and Teaching (B-EDS 122). Jerome Bruner’s theory has direct implications on how you will teach mathematics.

The main ideas of Bruner’s theory can be summarised as follows:

- Learning is an active process. Learners select and transform information.
- Learners make appropriate decisions, think about what they are learning and test their own decisions as they learn.
- Learners use prior experience to fit new information into the pre-existing structures of what they already know.
- Scaffolding is the process through which able (knowledgeable) peers or adults offer support for learning. This assistance becomes gradually less frequent as it becomes unnecessary.
- Intellectual development includes three stages. The first, the enactive stage, refers to learning through actions – while working with concrete materials (e.g., counters, abacus, bottle tops or stones) and the use of objects. The second level, the iconic stage refers to the learner’s use of pictures or models. During this stage, learners make use of pictorial images in understanding the world. The highest level and the most abstract, the symbolic stage, is when mental manipulation of numerals takes place. At this stage, the ability to think in abstract terms develops.
- The notion of a spiral curriculum describes how a curriculum should revisit basic ideas, building on them until the student grasps the full formal concept.

Source: Rhalmi, (2016)

Your teaching of mathematics must start with concrete objects or experiences (from the known to the unknown). From direct/concrete experiences, learners will be able to construct abstract concepts e.g. time, mass, volume, etc.

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120 minutes

Refer to the module: Education Studies 2: Theories of Learning and Teaching, (B-EDS 122) and Unit 1, where Jerome Bruner’s theory is discussed. Also, read the summary above again.

1. Discuss the three stages of learning proposed by Bruner.
2. Why do you think young learners need to move through the three stages of learning as proposed by Bruner?
3. Give one example of LTSM you would use to teach number patterns at each stage of learning.
4. Do you agree with the following statement:

   “Mathematics is not a spectator sport”?

   Provide reasons for your answer and indicate how you will ensure that learners are actively involved when you teach mathematics? Write down at least three important things you can do to ensure this.

Commentary:
Although mathematical learning depends heavily on abstraction and generalisation, learners in the Intermediate Phase must be provided with many concrete experiences in a variety of contexts before they are ready to move towards more abstract processes. For example, learners need many concrete sharing experiences (with real objects) before they are ready for the semi-concrete level (working with models, pictures and drawings) and eventually use algorithms (quick methods) on an abstract level (working with numbers and symbols). This approach to the teaching of mathematics underlies the strategies that you will learn about in this module and the other four modules of Mathematics Teaching in the Intermediate Phase.

It is important to note that learners do not simply go through stages from concrete to abstract. They need to be actively involved in working with the different representations of all of the concepts that they learn about in the Intermediate Phase. For example, learners might be cope well doing abstract addition but will need to handle concrete objects when they start doing multiplication.

Also, the vital importance of connections between the concrete and abstract should not be underestimated. While you teach you need to make those connections – speak about how, for example, that counting all the blocks as you put them in a bag, gives you a way to find out how many blocks you have in total. Then write down the addition you have done symbolically on the board. Show the class how, what you write in symbols, links to what they have done concretely so that they develop the ability to move from concrete to semi-concrete to abstract.

In the next series of three diagrams, we provide you with more information on each of the levels (concrete, semi-concrete, and abstract) in learning mathematics. These levels are applicable for Foundation Phase as well as Intermediate Phase. Make sure you have a clear understanding of each of the levels and that you know how to make the connections between the levels – this will help the learners to move between them.
All of the illustrations below, since they are not real objects, are semi-concrete. When you work concretely with learners, it means you are allowing them to hold, feel, look at, and work with REAL objects. The drawings in this manual are to show you the kinds of things that you could allow learners to work with. You cannot replace a concrete object with a picture, at the concrete level. Once you move to work with drawings, pictures or diagrams, you have moved to the semi-concrete level. These drawings help to make a bridge between the concrete and the abstract. Some drawings are more specifically structured than others, to help make the links between concrete and abstract. You, the teacher, will need to help learners make the connection by talking to them about the links.

Briefly, the examples below take you through the three stages of teaching the number concept. For example, the concept of ‘5’ would involve firstly, counting 5 real things (objects), then counting five drawings/pictures of things, or 5 moves on a number line and finally reading and writing the number symbol ‘5’. (Go back to Unit 1, Section 1, paragraph 3.3 in the CLG if you cannot remember the enactive, iconic and symbolic modes of Bruner.)

**LEVEL 1: CONCRETE STAGE**

(Working with real objects, e.g. fingers, counters, etc.)

During the concrete stage, learners’ thinking is limited to the use of **real objects**. They use their bodies and their senses to learn number concepts. They need to be physically involved and see, hear, touch, smell and even taste real objects.

The first concrete learning materials are the parts of the learner’s own body. In learning how to count, for instance, learners can use their fingers, toes or other body parts. Gradually move them away from using fingers and body parts, and let them use objects such as stones, bottle tops or any other counters instead. Once learners have mastered counting and calculating with concrete objects, they are ready to progress to the semi-concrete level and work with slightly more abstract resources.
LEVEL 2: SEMI-CONCRETE STAGE
(Working with representations, e.g. drawings, pictures, dominoes, dice, etc.).

During the semi-concrete stage, learners' thinking is limited to representations of real objects. This means that they do not have to use concrete objects to solve a Mathematical problem any longer, but they still need to use things that represent (stand for) the real objects, e.g. drawings or other 2-D materials. The semi-concrete stage is often also called the two-dimensional or pictorial stage.

By counting objects in pictures or dots on dominoes, learners begin to realise that it is possible to represent actual numbers or objects with symbolic things (e.g. dots or drawings).

Once learners have had sufficient experience with using concrete as well as semi-concrete resources (LTSM), they are ready to work with numbers to do calculations. When learners use symbols (numbers) to do calculate, they have reached the abstract stage.

LEVEL 3: ABSTRACT STAGE
(Working with symbolic representations, e.g. words, numbers).

The abstract stage is the stage where symbols represent concrete objects, e.g.

One apple – concrete object. 1 one
Number symbol (1); Number name (one).

Learners learn that letters and numbers have meaning – that they represent something. At the abstract stage learners can think without using concrete or semi-concrete materials and can count and solve problems mentally (in their minds). For example: They do not have to physically put together and count three counters and two counters to find the answer; they can just add mentally (3 and 2 is 5) or solve a problem like: 3 + 2 = 5.
From the series of diagrams, you could see that young learners learn progressively from the concrete to the abstract. Even in the Intermediate Phase, the order: **From the concrete, through the semi-concrete to the abstract**, is still applicable. For you to plan meaningful learning experiences in mathematics, you must keep these stages in mind. For every concept that you teach, you should enable them to move from the concrete to the abstract. This is done by allowing learners to experience activities at the three levels until they show they have reached level 3 (abstraction).

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Prepare yourself thoroughly so that you can actively participate in the group discussion/activity during the next student academic support session or work with a partner in your own study group.

1. Considering the THREE levels of learning mathematics. Develop three teacher-guided activities to show how Grade 4 learners can be involved at each of the levels (concrete, semi-concrete, and abstract) in a mathematics lesson when teaching a new topic. Work in small groups and clearly indicate the following in your activities:
   - Topic;
   - Suitable Objective(s);
   - LTSM required for each level; and
   - Active learner involvement on each level.

2. Be ready to present your activities or part thereof to the group to get their feedback.

3. After presenting your lesson to the group evaluate the lesson plans prepared. Complete a copy of the checklist below and give constructive feedback in the **Comment** column.

   Note: Comments should not be limited to aspects that have been ticked off under the **No** column. Even a **Yes** comment must be substantiated.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Yes</th>
<th>No</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>The lesson has a suitable mathematics topic.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clear objective(s) that speak to the topic has been formulated.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Appropriate LTSM has been listed for the different levels of learning.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### 3.4 Peer-assisted learning

Sometimes learners are more spontaneous, relaxed and comfortable amongst peers and learn best in such situations because they do not feel threatened. Learners who did not grasp a concept can be paired or grouped with peers who have a clear understanding of a concept. They will benefit from their peers’ support. In such a situation knowledge is presented on an as needed basis as opposed to a predetermined sequence of the teacher’s presentation. An important part of peer-assisted learning is to allow the learners who do not comprehend, an opportunity to explain his/her understanding of the peer(s). In this manner, an opportunity is provided to determine such learners’ understanding and/or misconceptions about a particular concept (van de Walle et al., 2014). In peer-assisted learning, most learners are not scared to ask and offer explanations that make sense to them on their own level.

In the next section, assessment in mathematics, as an integral part of teaching and learning, will be discussed.

### SECTION 4: ASSESSMENT IN MATHEMATICS

#### 1. INTRODUCTION

The Curriculum and Assessment Policy Statement for Mathematics in the Intermediate Phase (DBE, 2011, p. 293) describes assessment as follows:

“Assessment is a continuous planned process of identifying, gathering and interpreting information regarding the performance of learners, using various forms of assessment. It involves four steps: generating and collecting evidence of achievement, evaluating this evidence, recording the findings and using this information to understand and thereby assist the learner’s development in order to improve the process of learning and teaching.”
90 minutes

Reflect on the definition of assessment provided in the Curriculum and Assessment Policy Statement for Mathematics (DBE, 2011) to answer the questions:

1. In your own words, explain your understanding of the term **assessment**.
2. Why do we assess? What are the reasons for assessment – especially in mathematics?
3. How were you assessed in mathematics when you were at school? Which forms of assessment were used?
4. Do you think a variety of sources and techniques (involving different forms of assessment) should be used when assessing learners’ mathematics knowledge and skills? Provide reasons for your answers.
5. According to van de Walle, Karp and Bay-Williams (2014, p. 35) assessment is part of lesson planning and closely linked to lesson objectives:
   
   “You might wonder why you are thinking about assessment before you have even introduced the lesson, but thinking about what it is you want students to know and how they are going to show that to you, is assessment.”

   Do you agree with this statement? Why or why not? Provide reasons for your answer.

Assessment must never be seen as something **done at the end** of teaching and learning. It is an integral part of teaching and learning that must be already kept in mind when planning for teaching and learning. You will learn much more about the process and principles of assessment in the module: Education Studies 3: Curriculum, Pedagogy and Assessment (B-EDS 213), in Year 2. For now, let us look at the reasons for mathematics assessment.

### 2. REASONS FOR ASSESSMENT

When you assess your mathematics learners, you need a clear understanding of the reasons for assessment; or else, your assessment will be meaningless. We will now discuss the important reasons for assessment.

#### 2.1 Determine a learner’s level of development

As an Intermediate Phase teacher, you are required to know your learners and their levels of development. This is important because you need to:

- Accommodate learners’ needs and interests in your planning and mathematics teaching.
• Implement early identification of barriers/challenges.
• Implement appropriate learning support and intervention with regards to learners’ barriers/challenges.
• Know when (and how) to adapt your mathematics teaching and assessment.

Through assessment, you will be able to identify your learners’ strengths and weaknesses. You ought to find ways to improve the learners’ weaknesses and build on the learners’ strengths so that their mathematics confidence can improve for them to gain the courage to try new things and want to learn more about mathematics.

2.2 Enable meaningful guidance and assistance to learners

Without authentic (true/realistic) mathematics assessment, you will not be able to provide your learners with meaningful guidance and assistance. For this reason, you need to assess your learners on an ongoing basis. If you assess your learners continuously, you will know exactly how learners are progressing, what their challenges in mathematics are, and what guidance and assistance they need to help them overcome the challenges.

2.3 Provides information to give feedback to parents

The school and home environment should be complimentary. As a mathematics teacher, you need to give regular feedback to parents on learners’ mathematical progress. You also need to provide educational guidance and assistance to learner’s parents regarding the education of their child. Your approach to guidance must be respectful and sensitive. You should never patronise (belittle) parents. The ideal situation is that you and the learner’s parents should work together to overcome any learning challenges that learners may experience.

Your feedback on a learner’s mathematics assessments will equip parents to support learners in constructive ways. Moreover, the parents will become active participants in their child’s education.

2.4 Help to improve teaching

Teachers, who are the most effective, prepare thoroughly and systematically evaluate their own work (Gordon & Browne, 2011, p. 194). Evaluating one’s own work involves being aware of oneself and your own performance, one’s learners and their progress, and the teaching and learning environment.

A very important reason for the assessment of learners is to help teachers to improve their teaching. It is, therefore, necessary that you reflect on your own mathematics teaching and assessment skills and methods regularly. This involves not only
conducing but also analysing the assessment results of your learners. For example, look at which assessment activities your learners performed well in and in which assessment activities your learners experienced difficulties or challenges. From here, you should analyse your findings honestly and objectively. This should guide you, as a mathematics teacher, to reflect on how you can change or adapt your teaching or assessment of learning. Your reflection needs to include questions on whether the assessment strategies you are using are developmentally appropriate in terms of the learners and appropriate to what you are assessing. Such self-reflection will enable you to obtain information about the quality of your own mathematics teaching and assessment.

Now that you have gained information about the reasons for assessment, you need to learn more about WHAT must be assessed in mathematics learning.

3. WHAT TO ASSESS IN MATHEMATICS

The current mathematics curriculum guides you as to when you should teach which concepts and skills to learners in the Intermediate Phase and Grade 7. This again will guide you on what concepts and content should be assessed in which term of each grade. Keep in mind that learners need adequate time to gain the required knowledge and to acquire and practice their skills before you can assess them.

The weighting of mathematics content areas as prescribed in the mathematics curriculum (as discussed in Section 1, of Unit 2) guides the mathematics teacher regarding the time needed to adequately address the content within each content area in the classroom; it also provides guidance on the spread of content to be included in summative assessment (especially the end-of-the-year examination).

In the Intermediate Phase (Grades 4 to 6), the weighting of the mathematics content areas is the same for each grade in this phase; however, you will note that for Grade 7 the weighting is different. Do you know why? In Grade 7, Numbers, Operations and Relationships; Patterns, Functions and Algebra; as well as Space and Shape have nearly equal weighting. These requirements must form the base for teaching, learning and assessment. It can be used as assessment criteria when planning assessment tasks for your learners.

<table>
<thead>
<tr>
<th>Content area</th>
<th>Grade 4</th>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
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<tbody>
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<td>50%</td>
<td>50%</td>
<td>50%</td>
<td>30%</td>
</tr>
<tr>
<td>Patterns, Functions and Algebra</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>25%</td>
</tr>
</tbody>
</table>
**Space and Shape (Geometry)** | 15% | 15% | 15% | 25%
---|---|---|---|---
Measurement | 15% | 15% | 15% | 10%
Data Handling | 10% | 10% | 10% | 10%
---|---|---|---|---
| 100% | 100% | 100% | 100%

---

**60 minutes**

Study the weighting of mathematics content areas given above.

1. Why do you think the weighting of the content area Numbers, Operations and Relationships are so much higher than the other content areas in the Intermediate Phase?

2. Discuss the weighting of content areas prescribed for Grade 7 as opposed to the weighting in the Intermediate Phase. What do you notice? Why do you think there is a difference?

**Commentary:**
Did you think of the importance of Numbers, Operations and Relationships for the Intermediate Phase and how this content area can assist learners to become numerate? Can Data Handling or Measurement, for example, be taught without involving numbers? Think of examples of how numbers and number relationships feature in learning about handling data. If learners do not have a very good grasp of Numbers, Operations and Relationships, do you think they will be adequately prepared for mathematics in the other content areas of mathematics?

What is the focus of mathematics in the Senior Phase? Is this in line with the weighting given for Grade 7? How does the content area of Patterns, Functions and Algebra prepare learners to do algebra in the Senior Phase?

To be an assessor is one of the collective roles of teachers in a school (DHE, 2015, pp. 60-61). You, as a mathematics teacher, should understand that assessment is an essential part of the teaching and learning process therefore you need to know how to integrate assessment into this process. You, need to know the assessment criteria that the learners need to meet in each of the grades. As a teacher, you should know how to assess, and how to plan for assessment activities so that every learner has an opportunity for success. This will be discussed in detail in the other four modules of Mathematics Teaching in the Intermediate Phase.
As discussed in section 2 of this unit, mathematics assessment should be continuous. Different forms of assessment must be used and you need to give your learners multiple (more than one) opportunities to show whether they have mastered the prescribed mathematical concepts, content and skills. Assessment can take a variety of forms and teachers often need to use different activities and forms of assessment to assess the same knowledge and skills.

Before we start planning for mathematics assessment, we need to differentiate between informal daily assessment and formal assessment. Both of these have a role to play in learners' mathematics assessment.

### 3.1 Formative assessment

Formative assessment (sometimes also called informal assessment) is a daily form of assessment and is used to track the learner's progress during the development of concepts or skills. It should provide the learner, parents and teacher with feedback. The informal assessment helps the teacher to plan for teaching and learning. The CAPS document does not indicate if the informal assessment should be recorded. It is the decision of the teacher in the class.

Examples of informal assessments are written exercises, oral activities and presentations, written class tests and other forms of assessment. Informal assessment is done during day-to-day activities the learners perform in class. A typical example of an informal daily assessment is daily mental mathematics tests. Through these informal daily assessments, learners' mental mathematics skills are developed and improved daily.

To control and monitor these mental tests, mathematics teachers do not have to assess or mark informal daily assessments themselves. This is an ideal opportunity to allow for self- and peer-assessment. In this way, learners learn from and can reflect on their own performance.

Although some mathematics teachers may think that peer and self-assessment do not have much value, there is evidence that self- and peer-assessment can be extremely valuable in mathematics teaching and learning. The next reading activity focuses on peer and self-assessment in mathematics.
Read the following extract from the book: *Language for Learning Mathematics – Assessment for Learning in Practice, Chapter 4* (Lee, 2006, pp. 61-62) from your core reader, and answer the questions that follow.

**Extract:**

“**Peer and self-assessment**

Peer assessment and self-assessment are important forms of assessment that engage pupils in talking about their learning and therefore help them to become self-critical and independent. They are not replacements for teacher marking and feedback. As with all areas of learning mathematics, pupils need to learn how to talk about their learning, the language and expressions to use in critiquing others’ work and how to discuss problems and strategies with one another. This will take time and effort but the rewards are worthwhile. The pupil has to be an active agent in appraising and then closing the gap between their understanding at the outset and the objective of the learning, which means that effective learning must involve self-assessment by the pupil.

The act of talking about ideas and concepts makes those ideas available for feedback, from the teacher, from peers or the pupils themselves. Provided that feedback is about the learning objective, identifies what has been done well and sets out ways to improve, it will help pupils to learn more. Peer assessment helps pupils learn the skills of self-assessment and also provides a rich resource of ideas that pupils can use in their own learning. When a group of pupils is engaged in peer assessment, the group will be talking about mathematical ideas and making and sharing meanings about those ideas. Each member will be part of the group’s articulation of ideas and strategies and will thereby begin to internalize both the language and the ideas that are used. Peer and self-assessment provide a framework for talking about learning and therefore encourage meta-cognition, that is, thinking and talking about how and what pupils are learning.

Engaging in peer and self-assessment enables pupils to become self-reliant learners; they can guide their own learning because they know what they are trying to achieve and what they have to do to get there. Through peer and self-assessment, pupils become involved in the analysis and constructive criticism of their own work and this increases their rates of progress and levels of attainment. Pupils become able to focus their learning on the areas in which they feel they have the least confidence. They can pinpoint which parts/concepts in the topic give the most difficult and concentrate their efforts where it will help most. Peer and self-assessment also enable teachers to learn more quickly and accurately about their pupils ‘ideas or difficulties and give them a deeper understanding of all pupils’ progress and problems. Teachers can then...
decide where their time is best employed, who can carry on and who needs particular input.

All pupils can be involved in peer and self-assessment. Even in schools for pupils with learning difficulties I know pupils who think about one another’s work and give their peers feedback and thereby learn a great deal about their own work. Pupils studying mathematics at a high-level value collaboration in what can sometimes seem a lonely enterprise. They want to learn more and know that they can sort out problems together. Peer assessment can be a useful tool to help pupils begin to use specific phrases and ways of expression, as they need them to feedback to one another.

Pupils may need to learn ways to appreciate their own progress as many pupils undervalue their own work. Peer assessment will help them develop a more accurate view of their own abilities. Most pupils are honest in their assessment of their own work, most of the time. However, some pupils do not like to admit that they are not coping and say they understand when they do not, and peer and self-assessment are important ways to help overcome this. Pupils need constant reassurance that they are learning, and that, when work appears difficult, this is when they are learning the most. Pupils are often more honest and challenging with one another than the teacher would be or they would be to the teacher. I know of a pupil that tidied up his work instantly when a peer told him that it was unreadable. The newly-organized notebook revealed a great deal more about the pupil’s mathematical ability to both the teacher and himself. Pupils challenge one another more than a teacher would feel able to do. When they know what is possible, what they are aiming for and how to get there they can be very demanding of one another. The whole process enables pupils to become more objective about their own work and to build up an idea of the quality of work that they can achieve.”

**Questions:**

1. How can self and peer assessments enhance mathematics learning? From the above extract, identify ten ways in which peer and self-assessment can promote mathematics learning.
2. How and when will you make use of peer and self-assessment during mathematics teaching? Motivate your answer.

Assessments become formative assessments when the information provided from assessments is used to adapt the teaching work to meet learning needs (Black, Harrison & Lee, 2003). Formative assessment, also known as assessment FOR learning should include activities on different cognitive levels. The information gathered during formative assessment tasks, such as oral questions posed in class, homework activities, group activities and short class tests, informs teachers about learners’ understanding as well as challenges and teaching strategies should be adapted accordingly for future learning. Furthermore, the informal assessment provides opportunities for learners to prepare themselves for summative assessment.
You should also let your learners write weekly revision tests to reinforce and revise work done during the week. Weekly revision tests help learners to overcome test anxiety. In this way, learners get used to writing mathematics tests. It is like in the old saying: **Practice makes perfect.** Writing weekly revision tests also allows you to identify problem areas and provide appropriate learner support.

Short daily class tests can be used as a tool to monitor learners’ progress. Daily class tests should not take more than 10 minutes to conduct. Each class test must be based on work done in the previous lesson. It can be done orally, or you can write the test on the board. Learners can only write answers to the questions. They can mark their own work or peer marking can be applied.

Let us now look at formal assessments in mathematics.

### 3.2 Formal assessment

Formal assessment for mathematics comprises of two components: School-Based Assessment (SBA) and end-of-year examinations. The results of these are used to show the learner’s progress by the end of a programme or term. Formal assessment tasks are marked and formally recorded by the teacher. This mark is used for the feedback on the report card at the end of a term. Formal assessments should be moderated for quality assurance purposes. It is used to determine the degree to which the learner has met the criteria the teacher intended the learner to meet during the term. This helps the mathematics teacher to know how well and what percentage of work the learner has mastered. It also serves as a reward system to the learner for meeting the criteria and is helpful to the teacher for planning assessment for the next stage or term of assessment.

The SBA component of formal assessment can take many forms. For Intermediate Phase mathematics, the following assessment forms are recommended: tests, examinations, assignments, investigations and projects. The Curriculum and Assessment Policy Statement (CAPS) for Mathematics Intermediate Phase (DBE, 2011, p. 294) indicates the **minimum requirements** for formal Intermediate Phase (Grades 4 to 6) mathematics assessment. These minimum requirements have been adapted for the interim (from 2020 onwards) until the new policies are in place (DBE, 2019a). Table 6 indicates the requirements for formal assessment for Mathematics Intermediate Phase. The focus content area (CA) is also indicated.
### Table 6: Requirements for formal assessment: Intermediate Phase Mathematics

<table>
<thead>
<tr>
<th></th>
<th>TERM 1</th>
<th>TERM 2</th>
<th>TERM 3</th>
<th>TERM 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of assessment tasks</strong></td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td><strong>Form of assessment</strong></td>
<td>Assignment, Test, Investigation, Examination, Project, Test, Examination</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Marking guideline</strong></td>
<td>Memo, Memo, Memo and/Rubric, Memo, Rubric, Memo, Memo</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Number of papers</strong></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td><strong>Minimum marks:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade 4</td>
<td>25</td>
<td>25</td>
<td>25 per paper</td>
<td>25 per paper</td>
</tr>
<tr>
<td>Grade 5</td>
<td>25</td>
<td>25</td>
<td>30 per paper</td>
<td>25 per paper</td>
</tr>
<tr>
<td>Grade 6</td>
<td>25</td>
<td>25</td>
<td>40 per paper</td>
<td>25 per paper</td>
</tr>
<tr>
<td><strong>Maximum Time allocation</strong></td>
<td>1 hour, 1 hour/paper, 1 hour, 1 hour/paper</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Term weighting (for reporting purpose)</strong></td>
<td>50% 50% 50% 50%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>SBA Weighting (75%)</strong></td>
<td>15%</td>
<td>11.25%</td>
<td>11.25%</td>
<td>15%</td>
</tr>
<tr>
<td><strong>End of the year examination (25%)</strong></td>
<td>Paper 1: CA 1, Paper 2: CA 2, 3, 4, 5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Content areas covered</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade 4</td>
<td>CAPS: p. 35-65</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade 5</td>
<td>CAPS: p. 123-153</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade 6</td>
<td>CAPS: p. 213-238</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: DBE (2019a, p. 52)

The minimum requirements for Grade 7 mathematics assessment are as follows:
## Table 7: Requirements for formal assessment: Senior Phase Mathematics

<table>
<thead>
<tr>
<th></th>
<th>TERM 1</th>
<th>TERM 2</th>
<th>TERM 3</th>
<th>TERM 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of assessment</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>tasks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Form of assessment</td>
<td>Assignment</td>
<td>Test</td>
<td>Investigation</td>
<td>Examination</td>
</tr>
<tr>
<td>Marking guideline</td>
<td>Memo</td>
<td>Memo</td>
<td>Memo and/ Rubric</td>
<td>Memo</td>
</tr>
<tr>
<td>Number of papers</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Minimum marks: Grade 7</td>
<td>50</td>
<td>40</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>Maximum time allocation</td>
<td>1 hour</td>
<td>1.5 hours / paper</td>
<td>1 hour</td>
<td>1.5 hours / paper</td>
</tr>
<tr>
<td>Term weighting (for reporting purpose)</td>
<td>40%</td>
<td>60%</td>
<td>40%</td>
<td>60%</td>
</tr>
<tr>
<td>SBA Weighting (40%)</td>
<td>8%</td>
<td>6%</td>
<td>6%</td>
<td>8%</td>
</tr>
<tr>
<td>End of the year examination (60%)</td>
<td>6%</td>
<td>6%</td>
<td>6%</td>
<td>6%</td>
</tr>
<tr>
<td>Content areas covered</td>
<td>Paper 1: CA 1, 2</td>
<td>Paper 2: CA 3, 4 where applicable</td>
<td>Paper 1: CA 1, 2</td>
<td>Paper 2: CA 3, 4, 5</td>
</tr>
<tr>
<td>Topics</td>
<td>Grade 7: CAPS, p. 38</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

All SBA tasks must be completed before the end-of-year examinations. You need to carefully plan to enable your learners to complete the required number of tasks per term. For this purpose, teachers need to include an assessment plan when they do their year planning for mathematics teaching and learning. They need to ensure that the tasks cover the subject content to achieve the broad aims of mathematics. When preparing the tasks, teachers need to ensure that the tasks cater to different cognitive levels e.g. knowledge, routine procedures, complex procedures and problem-solving.

All formal assessment tasks must be assessed by the mathematics teacher and should be moderated beforehand to ensure credible, fair, valid and reliable assessment. You are therefore advised to do thorough assessment planning and develop the assessment tasks for your learners well in advance. Some mathematics school textbooks for Grades 4 to 6 and Grade 7 offer developmentally appropriate examples of different assessment tasks covering a variety of forms of assessment – make full use of these if you have these available. NOTE: For the assessment to be
transparent, the assessment criteria for the assessment task should be provided to learners beforehand.

| 32 | 120 minutes |

Consider the minimum requirements given above for Intermediate Phase (Grades 4 to 6) mathematics assessment.

1. What is the difference between a mathematics test and an assignment?
2. Which content area in mathematics will be most suitable for a project? Why do you say so?
3. What is the ratio of the weighting between SBA assessment and end-of-year examinations for mathematics in the Intermediate Phase? Why does the SBA carry more weight than the end-of-year examinations in the Intermediate Phase?

Compare your responses to your peer at the student academic support session or in your study group.

In the next few paragraphs, you will learn more about each of the different forms of assessment for mathematics. You will also be given guidelines on how to develop your own mathematics-specific assessment tasks i.e. tests, examinations, assignments, investigations and projects. For each of the different forms of assessment, it is important to keep the specific mathematics content to be assessed and the cognitive development level of the learners in mind. Let us start the most traditional forms of assessment in mathematics i.e. tests and examinations.

### 3.2.1 Mathematics tests and examinations

Traditionally tests and examinations assess specific mathematical concepts and skills and are traditionally written (pen-and-paper) assessments. As suggested in the CAPS document for mathematics Intermediate Phase (DBE, 2011) various types of questions geared at different cognitive learning levels must be included in tests and examinations. The following is suggested:

- Knowledge (± 25%);
- Routine Procedures (± 45%);
- Complex procedures (± 20%); and
- Problem-solving (± 10%).

When you set a mathematics test or examination paper for your learners, you do not only have to keep the principles of assessment like fairness, validity, reliability and discrimination in mind (you will learn more about these principles in Education Studies
3: Curriculum, Pedagogy and Assessment, (B-EDS 213)) but you also have to think of the different cognitive levels of the questions that you plan to include. Mathematics tests and examinations must always be accompanied by a memorandum which indicates the marks allocation for each question in the test or examination.

In setting questions for mathematics tests and examinations, use Bloom’s taxonomy to address the different cognitive levels. This is helpful to ensure a balanced assessment task. Bloom’s (1956) research focussed on cognitive thinking levels and he classified thinking originally into five levels of complexities. Later, researchers refined his taxonomy to classify thinking into six levels:

![Figure 12: Bloom’s Taxonomy](source: Dreyer (2014))

The following is a brief explanation with mathematical examples of the different levels:

- **Level 1: Remembering** is the remembering of previously learned material. This only requires recall of specific information, such as to know terms, facts and procedures. For example, list the attributes of a rectangle.
- **Level 2: Understanding** means the understanding of new material, by constructing meaning from different types of functions. For example, find items that you can use to show the shape of a rectangle.
- **Level 3: Applying** takes place when the learner uses his knowledge in new situations. For example, draw a diagram or take a photograph of the shape (rectangle).
- **Level 4: Analysing** is the ability the learner has, to not only learn the information but also to understand how and why the information fits together. This is done by breaking the information up into parts to explore the
relationship. For example, Identify where the rectangular shape is found around the school.

- **Level 5: Evaluating** is reached when the learner makes his own judgement about information, ideas and materials. For example, explain why this shape (rectangle) is used in the places it is.

- **Level 6: Creating** is the highest order of thinking a person can have. It is the ability to put things together to form a whole or creating a new meaning or structure. For example, create an item that includes all the parts of the shape (rectangle). Draw and label your design.

It is important to keep these levels in mind when you plan for teaching, learning and assessment. Different action verbs can be used in questions to address the different levels of thinking.

A good mathematics test must be an appropriate and valid tool for measuring competence. A test for learners must:

- Have clear instructions;
- Focus on the knowledge, concepts and skills to be assessed;
- Be well balanced and include knowledge, application and higher-order questions;
- Be accompanied by a complete and detailed memorandum showing the allocation of the marks;
- Where necessary be moderated before it is written by a HOD or colleague who is a subject specialist;
- Be written under examination conditions;
- Be marked with constructive feedback, within a week;
- Be recorded (these records should be kept safe and be available so that learners’ progress can be seen); and
- Be followed up by remedial work or support if that is required.

Let us look at mathematics assignments next.

### 3.2.2 Mathematics assignments

The work included in a mathematics assignment can be more demanding and extended by nature, but should not be too long or too demanding for Intermediate Phase learners. The development level of your learners should still be kept in mind. Although it is also mainly an individualised assessment task (to be completed by an individual learner), resources may be used. Mathematics assignments may include questions from past tests or examination papers and could be completed at school or home. A mathematics assignment can also take the form of a problem-solving task. A memorandum or/and a rubric can be used to mark an assignment.
The following guidelines can assist you to set good mathematics assignments for your learners. A good mathematics assignment must:

- Focus on the knowledge, concepts and skills to be assessed according to the content covered but could include more than one concept;
- Be developmentally appropriate for the learners you are teaching (keep the interest and development level of your learners in mind);
- Have clear instructions for learners (learners must know exactly what is expected of them);
- Have resources available if learners need to make use of this;
- Promote the construction of mathematical knowledge and develop mathematical skills;
- Develop learners’ independence i.e. to work on their own and without the teacher’s direct supervision;
- Can be marked with a rubric but may also need a memorandum – especially if questions from past examinations are included; and
- Be followed up by remedial work or learner support if that is required.

Adapted from: Nel (2005)

### 3.2.3 Mathematics investigations

Some mathematics teachers struggle to develop suitable investigations for their learners because they may not be aware that mathematical investigations should promote critical and creative thinking. A mathematical investigation:

> “can be used to discover rules or concepts and may involve **inductive reasoning**, identifying or testing patterns or relationships, drawing conclusions, and establishing general trends” (DBE, 2011, p. 295).

Mathematical investigations allow learners to use different logical processes to make conclusions and justify suppositions (beliefs) and conjectures (assumptions). It often involves inductive reasoning in identifying patterns and relationships. A rubric is usually used to mark a mathematical investigation.

To develop your own mathematical investigations for your learners, keep the following guidelines in mind. A good mathematics investigation should:

- Have clear instructions;
- Stimulate logical, critical and creative thinking;
- Not only assess mathematical knowledge and skills but also mathematical communication and reasoning;
Help learners to construct new knowledge through inductive reasoning or deductive arguments;
Can involve the identification of a pattern, confirmation of a pattern/relationship, proof of a deductive argument or formulation of a conjecture (e.g. a formula, rule or theorem) and
Must be challenging but still allow learners to experience success.

Adapted from: Nel (2005)

Here is an example of an investigation given to a Grade 6 class. Follow the instructions and complete the investigation yourself before you go on. This one, based on number patterns, is not meant to be prescriptive to you in any way but may give you an idea of how to set your own investigations for your learners.

**INVESTIGATION: NUMBER PATTERNS**  
**GRADE 6**

**What you need:**
- Pen and paper

**What you must do:**
1. Write down the first ten odd numbers.
2. Add the consecutive odd numbers as follows: Write down the first one, then add the first two odd numbers, then add the first three odd numbers, etc. Write down the calculations and answers each time:
   - For example:
     - \( 1 = 1 \)
     - \( 1 + 3 = 4 \) etc.
3. Write the answers to the additions as a sequence. What type of numbers are in the sequence?
4. Write the numbers in the sequence differently. What do you observe?
5. From what you have learnt, formulate a conclusion (or rule) you can use to find the numbers in the sequence.
6. Without doing any further addition, use your rule to write down the sum of the first:
   - 6.1 Twenty odd numbers.
   - 6.2 Fifty odd numbers.
   - 6.3 Hundred odd numbers.
7. Write the answer of \( 13 \times 13 \) as the sum of consecutive odd numbers. Use any method to check and explain this answer.
1. Follow the instructions in the investigation above. Complete all the calculations in the investigation so that you can use it as a memorandum/guideline to mark learners’ work.

2. Does the investigation comply with the suggested requirements for investigations (motivate your answer in each case):

- Have clear instructions?
- Stimulate logical, critical and creative thinking?
- Not only assess mathematical knowledge and skills but also mathematical communication and reasoning?
- Help learners to construct new knowledge through inductive reasoning or deductive arguments?
- Involve the identification of a pattern, confirmation of a pattern/relationship, proof of a deductive argument or formulation of a conjecture (e.g. a formula, rule or theorem)?
- Is it challenging but still allow learners to experience success?

The teacher used the rubric below as a tool to assess the investigation. Now use the rubric to assess your own completed investigation.

<table>
<thead>
<tr>
<th>Assessment criteria</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Organisation and recording of data</td>
<td>No attempt to organize data – recording is muddled.</td>
<td>Some classification of data – minor errors in recording.</td>
<td>Correct and organised classification and presentation of data.</td>
<td>Well organised classification and labelled presentation of data.</td>
</tr>
<tr>
<td>Identification of patterns</td>
<td>No patterns are found.</td>
<td>Some patterns are identified but not satisfactorily described.</td>
<td>Patterns are correctly and sufficiently identified and described.</td>
<td>Patterns are correctly and sufficiently identified, described and effectively used.</td>
</tr>
<tr>
<td>Logic in mathematical reasoning</td>
<td>No logical reasoning.</td>
<td>Inconsistent analysis and interpretation. The reasoning is not clear.</td>
<td>Consistent analysis and interpretation. Reached a valid conclusion with</td>
<td>Consistent correct analysis and interpretation. Proper logical reasoning. Strong</td>
</tr>
</tbody>
</table>
INTRODUCTION TO MATHEMATICS TEACHING IN THE INTERMEDIATE PHASE  
CURRICULUM AND LEARNING GUIDE

<table>
<thead>
<tr>
<th>Mathematical language and communication</th>
<th>supporting arguments.</th>
<th>supporting arguments.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poor communication and incorrect mathematical language.</td>
<td>Some attempt to use correct language but ideas not communicated.</td>
<td>Correct use of mathematical language, clear explanations, good communication.</td>
</tr>
<tr>
<td>Correct use of mathematical language, clear explanations, good communication.</td>
<td>Excellent communication, correct and appropriate mathematical language, unambiguous and elegant explanations.</td>
<td></td>
</tr>
</tbody>
</table>

**TOTAL 20**

Take your completed investigation to the next academic support session or own study group meeting.

---

**120 minutes**

1. In your group, discuss the following regarding the investigation:
   1.1 Does the investigation comply with the suggested requirements for investigations? Motivate your answers.
   1.2 Did you respond in the same way to the questions? If not, try to explain your different responses.
   1.3 Are the instructions clear to learners? How would you improve on the investigation?
   1.4 Is the rubric a suitable instrument to assess learners’ responses or will a teacher also need a memorandum? Explain your answers.

2. With reference to the above investigation:
   2.1 Is this investigation suitable for Grade 6 learners? Explain your answer.
   2.2 How would you adapt the investigation for Grade 4 learners?
   2.3 How would you adapt the investigation for learners with special needs like a reading or a writing challenge?

3. Apart from number patterns, which other mathematics topics are integrated into the investigation. List two other topics.

4. Write down what you have learnt from completing the investigation, its assessment and from others in the group discussion. Keep your notes for future use during WIL or in your own teaching.

A mathematical project is quite different from the other forms of assessments for mathematics. In the next paragraph, you will learn more about mathematics projects.
3.2.4 Mathematics projects

In mathematics projects, learners must get the opportunity to apply mathematical knowledge and skills in real-life situations. In so doing, learners can practically experience mathematics as a human activity. Do you still remember that one of the specific aims of mathematics is to recognise that mathematics is a creative part of human activity? (See Unit 1).

A range of mathematical competencies and skills can be assessed through projects. In a mathematical project, learners need to “demonstrate their understanding of different mathematical concepts and apply it to real-life situations” (DBE, 2011, p. 295). As with investigations, the assessment criteria should be communicated and available to learners beforehand. Projects are normally done over some time and can be done in groups. Therefore, as a mathematics teacher, you will need to do careful and thorough planning beforehand.

When planning your own mathematics projects for your learners, keep the following guidelines in mind. A good mathematics project should:

- Be relevant – the school context, learners’ experience and interests must be kept in mind when identifying suitable topics for projects;
- Have clear instructions and be clearly explained to learners;
- Be completed over some time like two or more weeks (e.g. in Data Handling where the different stages can be completed over time);
- Focus on one content area of mathematics but may integrate other content areas;
- Not exceed five written pages and may include building or construction of concrete models;
- Preferably be completed by groups of 4 to 6 learners;
- Continuously be guided and assessed (in different developmental stages of the project) by the teacher although only the final assessment needs to be formally recorded; and
- Be finally assessed by the teacher using a rubric with clear and transparent assessment criteria.

Adapted from: Nel (2005)
1. Consider the scenario and then answer the questions:

**Scenario:**

The HOD at your school request you to explain to the new Grade 4 mathematics teacher why we need different forms of formal assessment in mathematics; what is the difference between the recommended different forms of assessment; and what to keep in mind when planning for mathematics assessment.

**Questions:**

1.1 Describe how will you explain to the new mathematics teacher why we need different forms of assessment in mathematics?

1.2 Draw a mind map to summarise the different forms of assessment to point out similarities and differences between the different forms of formal assessment.

1.3 Make notes of what you will keep in mind when you plan for assessment so that you can share this with the new teacher.

2. Choose any grade in the Intermediate Phase. Use the guidelines given above to plan a project for learners in Data Handling.

- Follow the guidelines for drawing up a mathematics project.
- Keep the curriculum requirements in mind.
- Choose a topic/name for the project.
- Write down the instructions to learners and what is expected of them in the completion of the project.
- Discuss how you will assess learners' projects.

3. Prepare a presentation based on the project that you have planned to present at your next academic support session or own study group. Ensure that you have followed the guidelines for good mathematics projects – if guidelines were not followed, consider revising your project before the next academic support session!
Allow each member to present his/her planned project. Discuss the following after each presentation:

- Were the guidelines for the good projects followed? Responses must be motivated.
- Are the instructions for learners clear? Explain why/why not.
- Does the assessment task have the potential to achieve its objectives? Provide reasons.
- Do you think Data Handling is a suitable area/topic to assess using the project? Why or why not? Which other mathematics content area/areas could be suitably assessed with a project? Give reasons for your answers.
- How can the assessment task be adapted for learners experiencing challenges/learning barriers?

Add suggestions from the group to your assessment task and keep it for future use during WIL or teaching. Learn from the tasks designed by other group members. Take notes for future use.

**Commentary:**
You will learn more about Data Handling and how to teach and assess Data Handling in the module: Mathematics Teaching in the Intermediate Phase 4 (I-MAT 424) in your fourth year.

After the assessment, the teacher must give feedback to learners, parents and other stakeholders.

### 4. FEEDBACK TO LEARNERS AND PARENTS

Feedback after the informal and formal assessment is vital – it is an important part of mathematics teaching and learning. It gives the mathematics teacher, parent and learner a clear view of the learner’s achievements during the term as well as the learner’s growth during the year. Constructive feedback can motivate learners and also serve as an acknowledgement of learner’s achievements. Constructive feedback should also suggest recommendations on how learners can improve on mathematics problem areas or content.

Feedback on mathematics assessments should have the following goals:

- To help the learners to maximise their learning strategies to improve achievement;
• To help the learners to become active participants in their own learning and to reflect on their own learning so that the learners can adapt their learning strategies;
• To recognise and praise the learner for achievements and the growth he/she shows in learning;
• To help the mathematics teacher to adapt his/her teaching styles so that every learner’s needs in the class can be met; and
• To ensure that assessment is an integrated part of mathematics teaching and learning (and not an added extra).

Mathematics teachers should use the continuous feedback from assessment activities to focus their planning for teaching, learning and assessment in a direction that will benefit the process of learning. Through continuous feedback, the teacher can help the learner to improve their learning. A teacher cannot mark a book/task and write “Good work” but only give the learner a “5” on the 7-point scale. The teacher should tell the learner how to improve on his skill, such as: “Well, done! You managed to complete the calculations correctly but need to spend a little more time on how you represent the answers to show that you have solved the problem completely.”

Mathematics feedback should be provided to learners continuously (on an ongoing basis) throughout the year. Every day in class, term report cards and interviews with learners and parents when necessary will help you to stay in contact with your learners and parents. This will help you to identify misconceptions at an early stage, address learners’ challenges and adjust your teaching to benefit the learning needs the learners to have and generally maximize mathematics learning in your class.

Feedback is one of the most important building blocks of assessment. Informed parents and learners will ensure better relationships between you and your learners and their parents. The learner’s progress will also be better because everyone is informed about what they need to do to help the learners to achieve to the best of their ability.

Read the extract from Lee (2006, pp. 59-60) and then answer the questions.

Extract:

“Once the feedback is given there is another step to take before it becomes formative: the pupil must act on or make a response to the feedback. Once again, this is a time issue. Without time to read and respond to written feedback pupils will not value it. Therefore, when teachers have ‘marked the books’ they should make sure that at some time during the next lesson, probably at the start, the pupils read and begin to respond to the improvement advice given.”
Depending on the comment, pupils might:

- check that they understand what has been written and make a note to respond as homework
- read the comment and respond immediately as the comment will take only a short time to act on
- read the comment and redraft work that has been started, improving the work in the way suggested
- read the comment and feel unsure about how to respond so decide to talk to a peer to make sure that they do fully understand
- read the comment and complete two or three similar problems to show that they have understood the suggestion made
- find the four errors that they made in their work and write a reply to the teacher that shows that they understand where they had been going wrong.

Giving pupils effective written feedback will always take more time than the conventional ‘ticking interspersed with random ‘goods’, as one girl called this style of marking. However, the conventional ‘ticking’-type marking has no learning function, whereas a comment that sets out what has been done well and how the pupil is to continue to improve will help pupils to learn mathematics more effectively. Remember that in a classroom that focuses on using language to learn mathematics, pupils will be receiving feedback much of the time. As they hazard an idea in class or group discussion they will quickly find out if it is a good idea or not and hear some other good ideas to use. As they talk through their own ideas with one another they will ‘hear’ their own thoughts and this is a valuable way to check that their ideas make sense. By thinking and talking about their learning, pupils regularly receive feedback from the teacher and their peers and receive it at the right time – when they are struggling to understand a concept. However, schools demand written feedback as well, which pupils need to receive when they complete a written task as this ensures that everyone receives considered advice on how to move their learning forward. Therefore, time must be found.

Teachers have devised many solutions to finding sufficient time to give effective feedback. These include:

- giving written feedback only every three weeks, but making sure every pupil gets some good advice when the feedback is given
- planning exactly which pieces of work should be given written feedback and which will be marked in class in some way; the work that will receive feedback from the teacher is often termed ‘key pieces of work’
- only spending time writing comments when the advice will make a difference to pupils’ learning, that is when the pupils will have the time and the opportunity to act on the comments given
- marking routine work in class and spending teacher time only on work that explores the learning that the pupil has been doing
- discussing ideas of quality with the pupils in class; part of this will involve looking at, assessing and commenting on the quality of one another’s work
Questions:
1. Which important step after feedback does the author refer to in the extract? Write an essay of 300 to 500 words to explain your ideas on how you will implement this step after giving mathematics feedback to learners.

2. Discuss your understanding of the conventional ticking-type marking as referred to in the above extract. In your opinion, does this type of marking have any value? Explain your answer.

3. Study the suggested solutions teachers have devised to find sufficient time to give effective feedback to learners. Evaluate each suggestion critically. Explain why, or why you will not be able to implement the specific suggestion in your own mathematics classroom.

5. SUPPORT TO LEARNERS

Learner support is a very important part of the assessment that is often neglected. Various learner support strategies can be used. These include working individually with learners, working with concrete material, additional homework, additional worksheets, additional support activities, support teaching, drilling, peer teaching and corrections. After the support has been given, learners must be reassessed to determine if they have mastered the concept that you have spent additional time on.

Something which is often neglected but you should take special note of is corrections. Corrections should ALWAYS be done after each test. Every assessment must be a learning opportunity for learners.

When you do corrections with learners, ensure that they know and understand HOW and WHY they need to correct their answers. Also, check learners’ corrections.

Do not simply read out the correct answers for the learners to write down. This has very little value and benefit for learners. Learners must know why they have something wrong and what they must do to get the correct answer. If you do not check learners’ corrections, they may reinforce the wrong concept.
Is it a good idea for teachers just to correct learners' work?

Reflect on what you have just read and how it might change your view on giving corrections for learners to do and managing this important aspect in your own classroom.

As you have seen in this unit, planning for each day is extremely important. It is therefore vital that teachers have a well-established understanding of mathematics and what it entails. In the Mathematics Teaching in the Intermediate Phase modules to follow, you will learn more about how to teach each content area of mathematics and give guidance to learners to avoid misconceptions.

To review what you have learnt so far, do the next activity.

1. Explain how the key works of the Constructivist theorists (Piaget, Vygotsky and Bruner) underpin each of the strategies reflected in the table below. Copy and complete the table in your workbook.

<table>
<thead>
<tr>
<th>Strategies to maximize learning</th>
<th>Key works of constructivist theorists</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actively involve learners</td>
<td>Piaget</td>
</tr>
<tr>
<td>Teach from the concrete to the abstract</td>
<td>Vygotsky</td>
</tr>
<tr>
<td>Peer-assisted learning</td>
<td>Bruner</td>
</tr>
</tbody>
</table>

2. Reflect on how you will create an inviting mathematics classroom for your Intermediate Phase learners. Do this by writing approximately 80 words on each of the following headings:

2.1 Clean and orderly classroom;
2.2 Mathematics corner;
2.3 Classroom display; and
2.4 Storage space.

In your discussion of each of the above aspects, make particular reference to why it will advance mathematics learning.
3. Read the scenario below, before answering the question that follows.

**Scenario:**

Mrs. Sithole teaches mathematics to Grade 4 learners. She always starts her lesson with a class test on the content covered in the previous lesson. She prefers to use the chalkboard only when explaining mathematics. Her explanations are restricted to a few pre-selected examples from the textbook, without considering her learners' developmental level. Learners' involvement is limited to copying the examples from the chalkboard into their workbooks. Most of the Grade 4 learners in Mrs. Sithole's class do not understand mathematics and therefore do not like the subject.

In a short essay of approximately 700 words, describe how you will advise Mrs. Sithole to improve her mathematics lesson. In your advice make particular reference to aspects such as:

- Teaching according to learners' developmental level;
- Actively involve learners;
- Teach from the concrete to the abstract;
- Peer-assisted learning; and
- Develop own mathematical knowledge.


- Draw a mind map to show how you can plan your formal mathematics assessment tasks for each term. Ensure that you cover the mathematics content to be assessed and for each assessment task per term, indicate the mathematics topic/content the task will focus on.
- In three paragraphs, explain how you did your assessment planning for the term. Include the factors you have considered for your planning.

6. In an essay of one to two pages, discuss the importance of providing constructive feedback and support to learners after both informal and formal assessments. Provide examples to support your arguments.

Reflect on what you have learnt in this unit by completing the self-assessment activity below. If your answer is UNSURE or NO on any of the criteria, go back to the relevant section to study it again.
### Self-assessment: Unit 2

<table>
<thead>
<tr>
<th>Now that I have worked through this unit, I can:</th>
<th>YES</th>
<th>UNSURE</th>
<th>NO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Define the aims of school mathematics.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Describe the mathematical skills embedded in the mathematics curriculum.</td>
<td></td>
<td></td>
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<tr>
<td>Outline the mathematics content areas and the time allocation for each.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use the curriculum to identify the mathematical content that must be taught in Intermediate Phase and Grade 7.</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Explain how to plan for mathematics teaching and learning</td>
<td></td>
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<td></td>
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<tr>
<td>Explain how to create an inviting mathematics classroom.</td>
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<tr>
<td>Describe strategies to teach and maximise learning in the Intermediate Phase classroom.</td>
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<tr>
<td>Plan for mathematics assessment in your Intermediate Phase classroom.</td>
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<tr>
<td>Implement different types of mathematics assessment in Intermediate Phase classrooms.</td>
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</tbody>
</table>

Congratulations, you have come to the end of the Module: Introduction to Mathematics Teaching in the Intermediate Phase (I-MAT 120). We hope that you have gained a lot from this module. In the next module the content area Numbers, Operations and Relationships will be discussed and how to teach it to Intermediate Phase and Grade 7 learners. The focus will be on pre-number concepts, number relationships, fractions, decimals, percentages and ratio and rate. Addition and subtraction of fractions and decimals are also addressed in the next module, i.e. Mathematics Teaching in the Intermediate Phase 1 (I-MAT 221).
REFERENCES


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Travelationship, 2010. *One of the teaching classrooms*. [Online], Available at: [https://www.flickr.com/photos/118889575@N05/13677600143](https://www.flickr.com/photos/118889575@N05/13677600143) [accessed 06 January 2020].


## ADDENDUM A: SANTS LESSON PLAN TEMPLATE FOR THE INTERMEDIATE PHASE

**SANTS Private Higher Education Institution**  
**GRADES 4, 5, 6 and 7 LESSON PLANNING FORM**

<table>
<thead>
<tr>
<th>NAME:</th>
<th>STUDENT NO.</th>
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</table>

**1. SUBJECT**

**2. TOPIC**

**3. DATE**

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<th>y</th>
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<th>m</th>
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<th>d</th>
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</tr>
</thead>
<tbody>
<tr>
<td>20</td>
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<td>-</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

**4. GRADE**  
(Mark the grade you will be teaching with an X)

**PLEASE NOTE THAT THIS LESSON PLANNING TEMPLATE IS AVAILABLE IN ELECTRONIC FORMAT ON MySANTS**

**5. NCS OUTCOMES/General aims (tick boxes)**  
Learners are able to:

- Identify and solve problems and make decisions using critical and creative thinking.
- Work effectively with others as members of a team, group, organisation and community.
- Organise and manage themselves and their activities responsibly and effectively.
- Collect, analyse, organise and critically evaluate information.
- Communicate effectively using visual, symbolic and/or language skills in various modes.
- Use science and technology effectively and critically showing responsibility towards the environment and the health of others.
- Demonstrate an understanding of the world as a set of related systems by recognising that problem-solving contexts do not exist in isolation.

**6. SUMMARY OF THE CONTENT TO COVER IN THIS LESSON**  
(Briefly summarise the content (see 7.2) that you will be presenting in this lesson)

**7. LESSON OBJECTIVE(S)**

**7.1 PRE-KNOWLEDGE**  
(Write down learners’ existing knowledge, skills and values)

At the start of this lesson the learners should already know… and can do…

**7.2 CONCEPTS and NEW KNOWLEDGE**  
(Write down the new knowledge, skills and values that you are going to teach)
### 7.3 LESSON OBJECTIVES

*Own lesson objectives based on General and Specific Aims from CAPS, written in your own words*

By the end of the lesson the learners should be able to...

### 7.4 FUTURE LEARNING

*Briefly describe what the learners will learn in the lesson that follows this one*

### 8. LTSM

*Name LTSM you intend using in this lesson and remember to reference ALL your resources (text books, websites, workbooks etc.) under Reference list of all sources consulted in point 9 below. Try to use a number of different items or types of LTSM.*

### 9. REFERENCE LIST OF ALL SOURCES CONSULTED

*List all the text books, workbooks, websites etc. that you used to prepare this lesson*

### 10. LESSON PHASES

#### 10.1 INTRODUCTION OF THE LESSON

*Time allocated: min*

*Give a detailed description of how you greet the learners, how you set the atmosphere for the new lesson, how you awaken the learners’ prior knowledge, and how you create a link between what the learners already know and the new knowledge that you will be presenting.*

#### 10.2 DEVELOPMENT – PRESENTING THE NEW KNOWLEDGE

*Time allocated: min*

*Give a detailed description of WHAT content you are presenting, HOW you will be presenting it, and WHAT THE LEARNERS WILL BE DOING.*
10.3 CONSOLIDATION
(Time allocated: min)
(Give a detailed description of how you will consolidate the new knowledge, as well as how you incorporate assessment of the objectives and how you will wrap up. Please also mention here any HOMEWORK that you will give the learners):

11. DIFFERENTIATION
11.1 LEARNER ENRICHMENT
(Indicate what measures are in place for learners who grasped concepts quickly. How will you challenge them and keep them from getting bored?)

11.2 LEARNER SUPPORT
(Indicate what measures are in place for learners who struggle to grasp the concepts. How will you support them and keep them from getting negative and frustrated?)

12. REFLECTION
Briefly reflect on your lesson by discussing its strengths (what went well), its weaknesses (what did not work), what you find challenging, if the lesson objectives were met and what you would improve if you had to teach this lesson again. Use the following questions to guide your reflection:

1. Describe aspects of your lesson that worked really well.
2. Which areas of your lesson did not go according to plan? Explain why you think this may have happened.
3. Re-examine your lesson objectives. Did you meet them? Why/why not?
4. This is what I learnt about the learners in my class today:
5. What was my most challenging moment in this lesson and why? How will I respond next time?
6. To what extent were the learners productively engaged in the learning process? Discuss.
7. If I had the opportunity to teach this lesson again to this same group of learners, what would I do differently? Why?
8. What evidence/feedback do I have that the learners achieved an understanding of the lesson objective(s)?